Constrain modified gravities with pulsar timing arrays

Zu-Cheng Chen (陈祖成)

Done with Yan-Chen Bi, Qing-Guo Huang, Jun Li,
Lang Liu, Zhu Yi, Yu-Mei Wu

Based on 2310.08366 (PRDL); 2401.09818 (PRDL); 2101.06869 (SCPMA);
2310.11238 (PRD); 2302.00229 (PRD); 2310.07469 (CQG)

湖南師範大學
HUNAN NORMAL UNIVERSITY

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Outline

Detecting a GWB with PTA

- Assume the GWB is isotropic, unpolarized, and stationary

$$\left\langle h_A^*(f,\hat{\Omega})h_{A'}(f',\hat{\Omega}')\right\rangle = \frac{3H_0^2}{32\pi^3 f^3}\delta^2(\hat{\Omega},\hat{\Omega}')\delta_{AA'}\delta(f-f')\Omega_{\rm gw}(f)$$

- Spectrum of GWB

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_{\rm crit}}\frac{d\rho_{\rm gw}}{d\ln f}, \qquad \rho_{\rm crit} = \frac{3H_0^2}{8\pi}, \quad \rho_{\rm gw} = \frac{1}{32\pi}\left\langle \dot{h}_{ij}(t,\vec{x})\dot{h}^{ij}(t,\vec{x})\right\rangle,$$

- Cross-power spectral density

$$S_{IJ} = \langle \tilde{r}_I^*(f)\tilde{r}_J(f')\rangle = \frac{1}{\gamma}\frac{H_0^2}{16\pi^4 f^5}\delta\left(f-f'\right)\Gamma_{IJ}(f,L_I,L_J,\xi)\,\Omega_{\rm gw}(f)$$

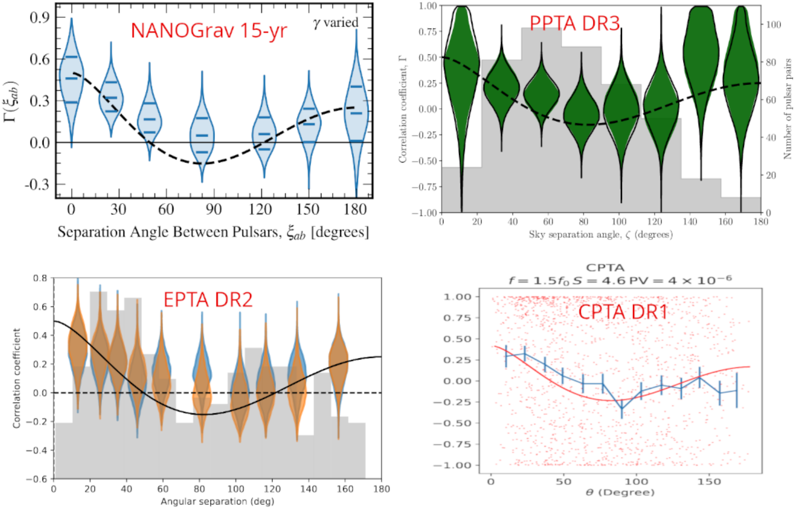
- Overlap reduction function (ORF) is function of f, L_I, L_J, ξ

$$\Gamma_{IJ} = \gamma\sum_A\int d\hat{\Omega}\left(e^{2\pi i f L_I(1+\hat{\Omega}\cdot\hat{p}_I)}-1\right)\times\left(e^{-2\pi i f L_J(1+\hat{\Omega}\cdot\hat{p}_J)}-1\right)F_I^A(\hat{\Omega})F_J^A(\hat{\Omega})$$

- Hellings-Downs correlations for $fL \gg 1$ (short-wavelength approximation)

$$\Gamma_{IJ} = \frac{3}{2}\left(\frac{1-\cos\xi}{2}\right)\ln\frac{1-\cos\xi}{2} - \frac{1-\cos\xi}{8} + \frac{1}{2}$$

Evidence for a GWB in PTA data sets



NANOGrav, 2306.16213 (ApJL); PPTA, 2306.16215 (ApJL)

EPTA+InPTA, 2306.16214 (A&A); CPTA, 2306.16216 (RAA)

SIGWs can explain the PTA signal.

NANOGRAV 15-YEAR NEW-PHYSICS SIGNALS 9

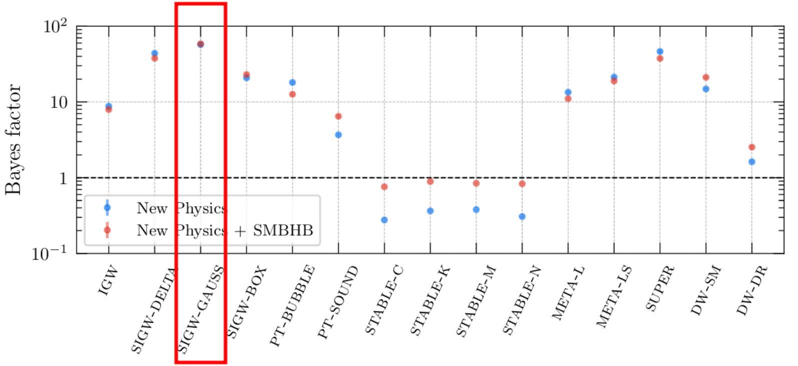


Figure 2. Bayes factors for the model comparisons between the new-physics interpretations of the signal considered in this work and the interpretation in terms of SMBHBs alone. Blue points are for the new physics alone, and red points are for the new physics in combination with the SMBHB signal. We also plot the error bars of all Bayes factors, which we obtain following the bootstrapping method outlined in Section 3.2. In most cases, however, these error bars are small and not visible.

NANOGrav Collaboration, 2306.16219 (ApJL)

Scalar-Induced Gravitational Waves (SIGWs)

- Primordial perturbations can be generated by quantum fluctuations during inflation.
- Metric perturbation in Newtonian gauge

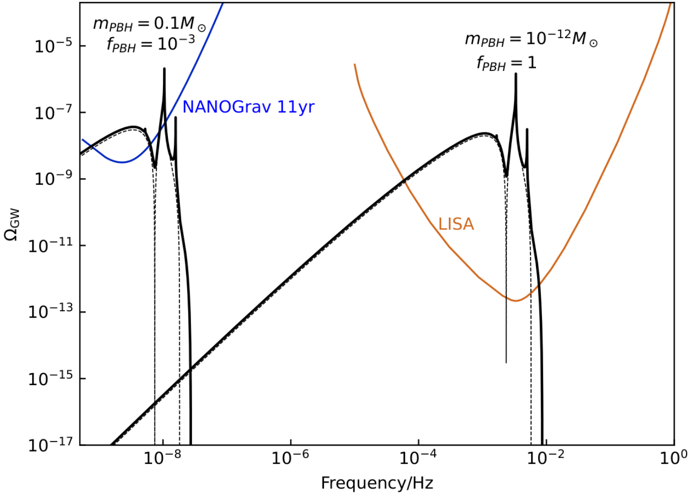
$$\mathrm{d}s^2 = a^2 \left\{ -(1+2\phi)\mathrm{d}\eta^2 + \left[(1-2\phi)\delta_{ij} + \frac{h_{ij}}{2} \right] \mathrm{d}x^i \mathrm{d}x^j \right\}, \tag{1}$$

where $\phi \equiv \phi^{(1)}$ and $h_{ij} \equiv h_{ij}^{(2)}$ are the scalar and tensor perturbations, respectively.

- Primordial scalar perturbations can generate SIGWs, as well as primordial black holes (PBHs).

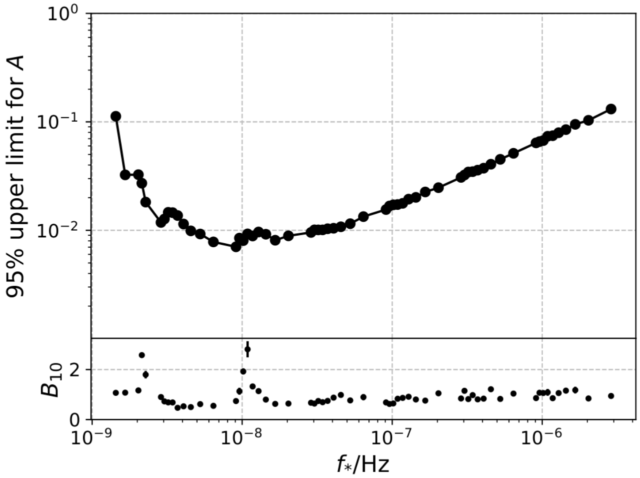
See also the talks from Fabrizio Rompineve, Guillem Domenech and Sonali Verma.

Detecting SIGW with PTA



*Chen Yuan, **ZCC**, Qing-Guo Huang, 1906.11549 (PRD Rapid Communications)*

Constrain SIGWs with NANOGrav 11-yr data set



ZCC, Chen Yuan, Qing-Guo Huang, 1910.12239 (PRL)

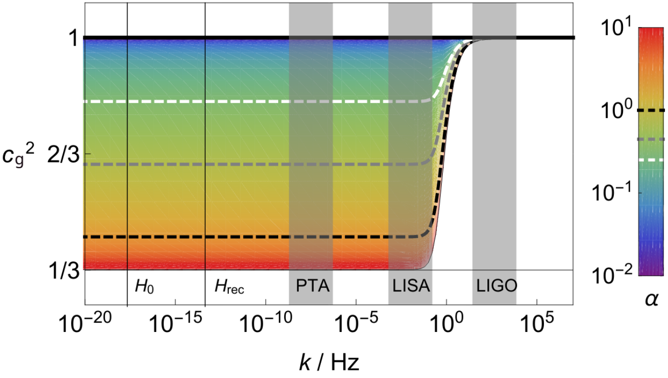
Question: Can we test gravity if the PTA signal is indeed from SIGWs?

Speed of GW

- GW170817: $-3 \times 10^{-15} \leq c_g - 1 \leq 7 \times 10^{-16}$

LVK, 1710.05832 (PRL)

- GW speed c_g can be frequency dependent



Claudia de Rham, Scott Melville, 1806.09417 (PRL)

Speed of SIGW

- EoM

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + c_g^2 k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta). \tag{2}$$

- SIGW spectrum

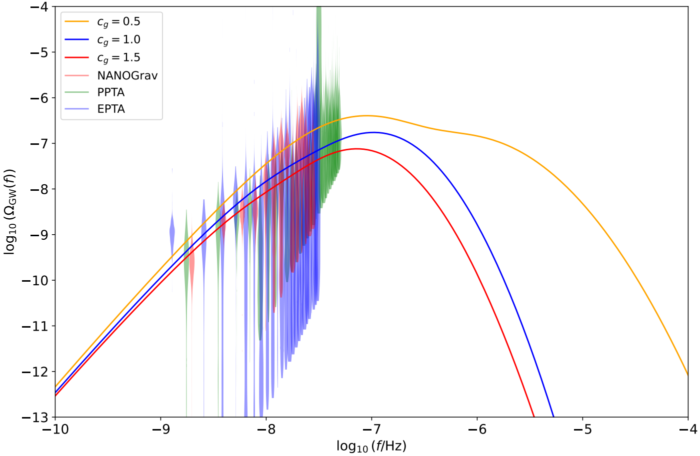
$$\Omega_{\mathrm{GW}}(k) = \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \mathcal{T}(u, v, c_g) P_\zeta(vk) P_\zeta(uk). \tag{3}$$

- Transfer function

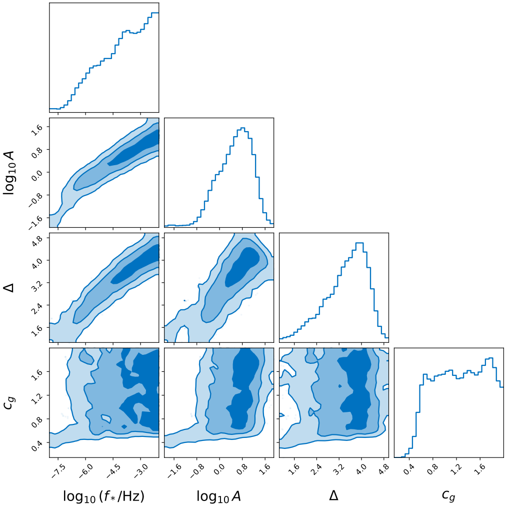
$$\begin{aligned} \mathcal{T}(u, v, c_g) = & \frac{3\left[4v^2 - \left(v^2 - u^2 + 1\right)^2\right]^2 \left(v^2 + u^2 - 3c_g^2\right)^2}{1024 v^8 u^8} \\ & \times \left\{ \left[\left(v^2 + u^2 - 3c_g^2\right) \ln\left(\left|\frac{3c_g^2 - (v+u)^2}{3c_g^2 - (v-u)^2}\right|\right) - 4vu\right]^2 \right. \\ & \left. + \pi^2 \left(v^2 + u^2 - 3c_g^2\right)^2 \Theta(v + u - \sqrt{3}c_g) \right\}. \end{aligned} \tag{4}$$

Jun Li, Guang-Hai Guo, 2312.04589

PE with NANOGrav 15-yr data set + PPTA DR3 + EPTA DR2



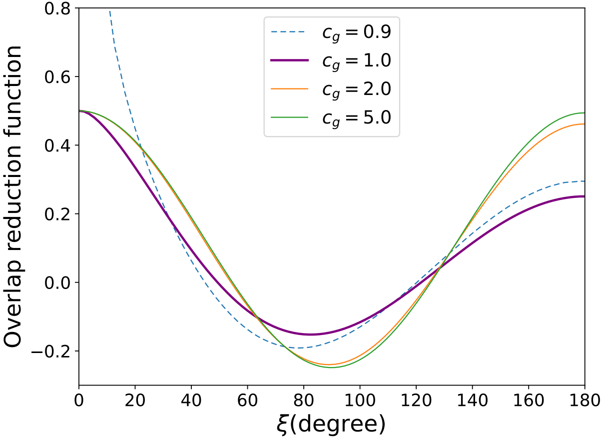
ZCC, Jun Li, Lang Liu, Zhu Yi, 2401.09818 (PRDL)



- $c_g \gtrsim 0.61$ at a 95% CI.
- Consistent with $c_g = 1$.

ZCC, Jun Li, Lang Liu, Zhu Yi, 2401.09818 (PRDL)

Overlap reduction function (ORF)

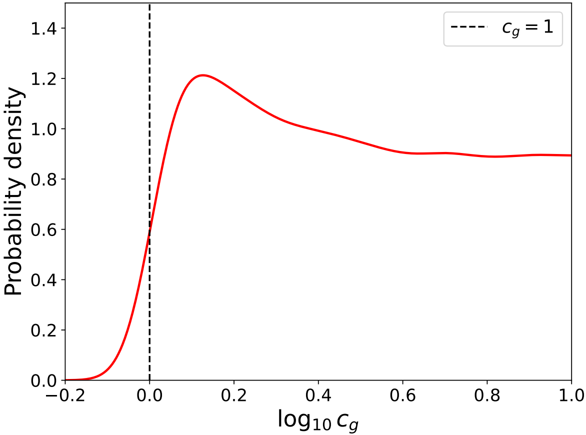


Reginald Christian Bernardo, Kin-Wang Ng, 2208.12538 (PRD)

Reginald Christian Bernardo, Kin-Wang Ng, 2302.11796 (PRDL)

*Yan-Chen Bi, Yu-Mei Wu, **ZCC**, Qing-Guo Huang, 2310.08366 (PRDL)*

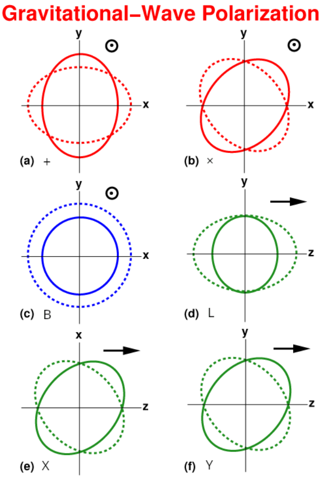
PE with NANOGrav 15-yr data set



- $c_g \gtrsim 0.85$
- Still consistent with $c_g = 1$.

*Yan-Chen Bi, Yu-Mei Wu, **ZCC**, Qing-Guo Huang, 2310.08366 (PRDL)*

Alternative polarizations



- A general metric gravity theory in $4D$ spacetime can have 6 polarization modes.
- polarization tensors
  - TT (Tesnsor Transverse) predicted by GR

$$\epsilon_{ij}^{+} = \hat{m} \otimes \hat{m} - \hat{n} \otimes \hat{n},$$

$$\epsilon_{ij}^{\times} = \hat{m} \otimes \hat{n} + \hat{n} \otimes \hat{m}$$

  - ST (Scalar Transverse)

$$\epsilon_{ij}^{B} = \hat{m} \otimes \hat{m} + \hat{n} \otimes \hat{n}$$

  - SL (Scalar Longitudinal)

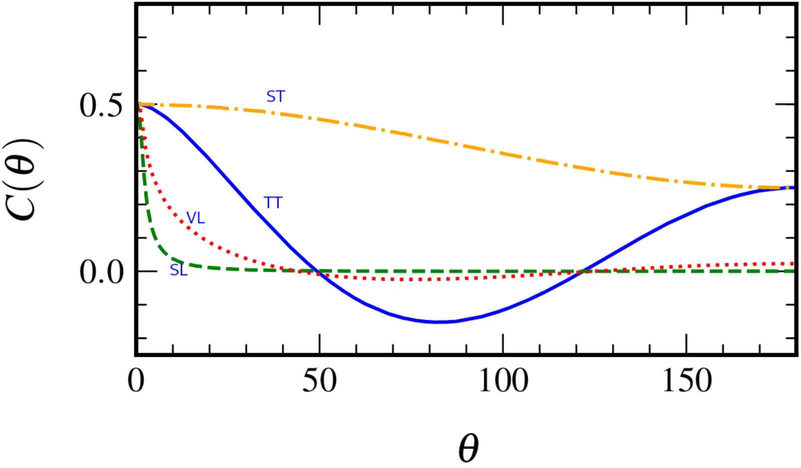
$$\epsilon_{ij}^{L} = \hat{\Omega} \otimes \hat{\Omega}$$

  - VL (Vector Longitudinal)

$$\epsilon_{ij}^{X} = \hat{m} \otimes \hat{\Omega} + \hat{\Omega} \otimes \hat{m},$$

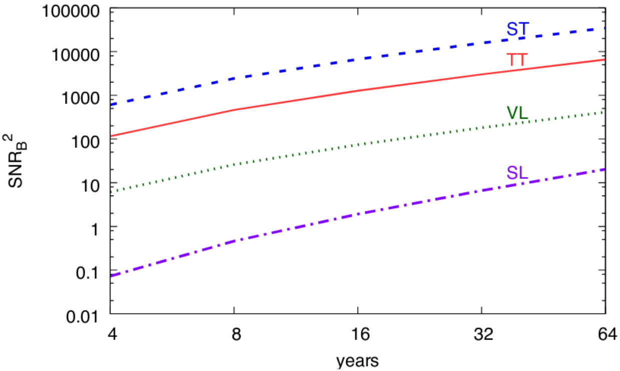
$$\epsilon_{ij}^{Y} = \hat{n} \otimes \hat{\Omega} + \hat{\Omega} \otimes \hat{n}$$

Overlap reduction function (ORF)



Spatial correlations: $|\Gamma_{ST}| > |\Gamma_{TT}| > |\Gamma_{VL}| > |\Gamma_{SL}|$

$$\mathrm{SNR}_B^2 = 2\sum_f \sum_a^{N_p} \sum_{b>a}^{N_p} \frac{\Gamma_{ab}^{I\,2}(f)}{\Gamma_{aa}^I(f)\Gamma_{bb}^I(f) + \Gamma_{ab}^I(f)}.$$



ST is the easiest to detect among the four polarization modes.

Neil J. Cornish, Logan O'Beirne, Stephen R. Taylor, Nicolás Yunes, 1712.07132 (PRL)

Evidence for the ST correlations in NANOGrav 12.5-yr data set

- Bayes factor compared to CRN model

  __ZCC__, Chen Yuan, Qing-Guo Huang, 2101.06869 (SCPMA)

Model	TT	ST	VL	SL	ST+TT
BF	4.96(9)	107(7)	1.94(3)	0.373(5)	96(3)

- No significant evidence for TT/VL/SL modes;
- Strong Bayesian evidence for ST correlations;
- No TT correlations in addition to the ST mode;
- The significance of ST signal is reduced when removing pulsar J0030+0451;

  NANOGrav, 2109.14706 (ApJL)

- The origin of the ST signal need to be further investigated.

Search for alternative polarizations in NANOGrav 15-yr data set

- Bayes factor compared to the TT model

  ***ZCC**, Yu-Mei Wu, Yan-Chen Bi, Qing-Guo Huang, 2310.11238 (PRD)*

Model	ST	VL	SL	GTb	TT + ST
BF	0.40(3)	0.12(2)	0.002(1)	3.9(3)	0.943(5)

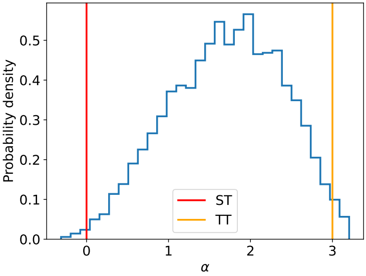
- No significant evidence supporting or refuting the ST model over the TT model; see also *NANOGrav, 2310.12138 (ApJL)*

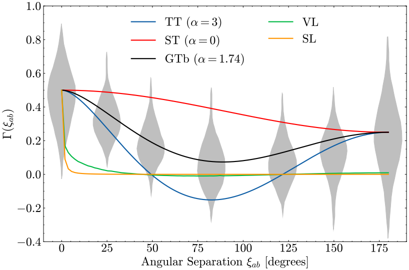
- VL and SL models are weakly and strongly disfavored, respectively.

We also consider a general transverse (GT) ORF parameterized as

$$\Gamma_{ab}(f) = \frac{1}{8}\left(3 + 4\delta_{ab} + \cos\xi_{ab}\right) + \frac{\alpha}{2} k_{ab} \ln k_{ab}. \tag{5}$$

ST: $\alpha = 0$ TT: $\alpha = 3$ prior of α: Uniform(-10, 10)

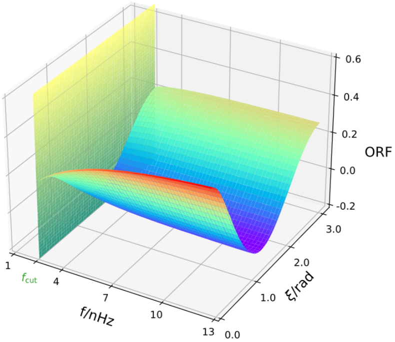




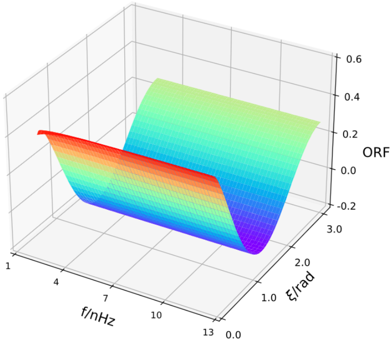
- Our analysis yields $\alpha = 1.74^{+1.18}_{-1.41}$, thus excluding both the TT and ST models at the 90% CL.

ZCC, Yu-Mei Wu, Yan-Chen Bi, Qing-Guo Huang, 2310.11238 (PRD)

ORF for massive gravity



$m_g = 10^{-23}$eV



$m_g = 10^{-24}$eV

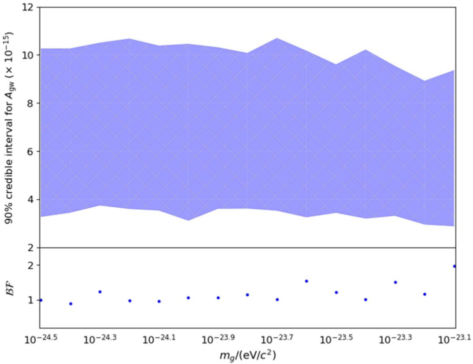
ORF is a function of graviton mass m_g, GW frequency f, and angular separation ξ.

Kejia Lee, Fredrick A. Jenet, Richard H. Price, Norbert Wex, Michael Kramer, 1008.2561 (ApJ)

Qiuyue Liang, Mark Trodden, 2108.05344 (PRD)

*Yu-Mei Wu, **ZCC**, Qing-Guo Huang, 2302.00229 (PRD)*

Constrain the graviton mass with the NANOGrav 15-yr data set



- It's challenging to distinguish the SGWB arises from massive gravity or massless gravity based solely on spatial correlations.
- The dispersion relation $\omega = \sqrt{m_{\rm g}^2 + |\mathbf{k}|^2}$ leads to

$$f_{\rm min} = \frac{m_g}{2\pi} \lesssim \frac{1}{T_{\rm obs}} \Rightarrow m_g \lesssim 8.2 \times 10^{-24}\,{\rm eV}$$

Yu-Mei Wu, ZCC, Yan-Chen Bi, Qing-Guo Huang, 2310.07469 (CQG)

Summary

PTAs are indispensable tools for testing modified gravity theories at nHz frequencies, including:

- Speed of GW: $c_g \gtrsim 0.85$

  *Yan-Chen Bi, Yu-Mei Wu, **ZCC**, Qing-Guo Huang, 2310.08366 (PRDL)*

  ***ZCC**, Jun Li, Lang Liu, Zhu Yi, 2401.09818 (PRDL)*

- Alternative polarizations: TT and ST both seem to be disfavored; unknown noise?

  ***ZCC**, Chen Yuan, Qing-Guo Huang, 2101.06869 (SCPMA)*

  *Yu-Mei Wu, **ZCC**, Qing-Guo Huang, 2108.10518 (ApJ)*

  ***ZCC**, Yu-Mei Wu, Qing-Guo Huang, 2109.00296 (CTP)*

  ***ZCC**, Yu-Mei Wu, Yan-Chen Bi, Qing-Guo Huang, 2310.11238 (PRD)*

- Massive gravity: $m_g \lesssim 8.2 \times 10^{-24}\,\mathrm{eV}$

  *Yu-Mei Wu, **ZCC**, Qing-Guo Huang, 2302.00229 (PRD)*

  *Yu-Mei Wu, **ZCC**, Yan-Chen Bi, Qing-Guo Huang, 2310.07469 (CQG)*

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Thank you for your attention!