

# Constrain Modified Gravities with Pulsar Timing Arrays

Zu-Cheng Chen (陈祖成)

Done with Yan-Chen Bi, Qing-Guo Huang, Jun Li, Lang Liu, Zhu Yi,  
Chen Yuan, Yu-Mei Wu

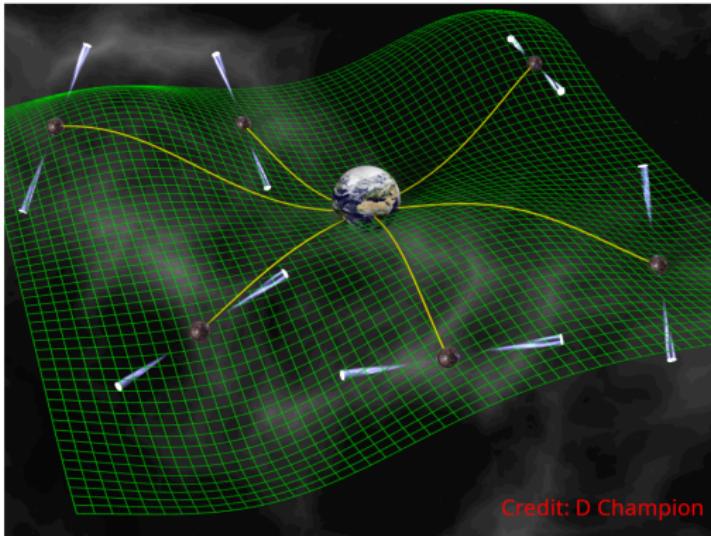
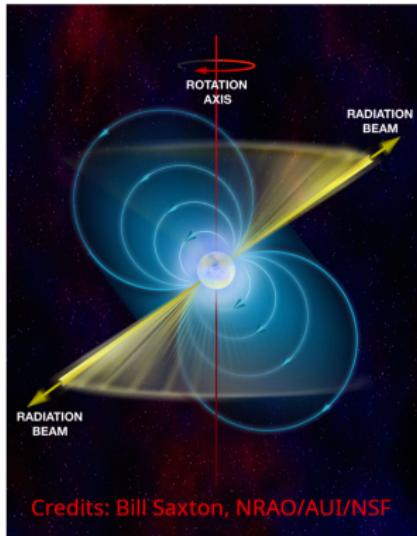
Based on 2310.08366 (PRDL); 2401.09818 (PRDL); 2101.06869 (SCPMA);  
2108.10518 (ApJ); 2109.00296 (CTP); 2310.11238 (PRD);  
2302.00229 (PRD); 2310.07469 (CQG)

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暨第六届伽利略-徐光启会议



湖南師範大學  
HUNAN NORMAL UNIVERSITY

# Pulsar and pulsar timing array (PTA)



- Pulsars are highly magnetized, rotating neutron stars that emit regular pulses of electromagnetic radiation.
- GWs can cause tiny distortion in spacetime inducing variations in the time of arrivals (ToAs).
- A PTA pursues to detect nHz GWs by regularly monitoring ToAs from an array of the ultra rotational stable millisecond pulsars.

## PTAs in operation

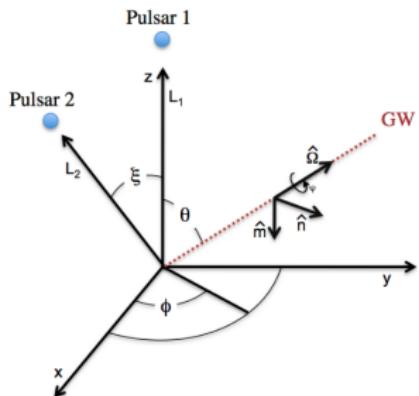


IPTA: PPTA + EPTA + NANOGrav + InPTA

Observers: CPTA, MPTA

# Timing residual induced by a GWB

- Redshift



$$\begin{aligned}
 z(t, \hat{\Omega}) &= \frac{\nu_e - \nu_p}{\nu_p} \\
 &= \frac{\hat{p}^i \hat{p}^j}{2(1 + \hat{\Omega} \cdot \hat{p})} [h_{ij}(t_p, \hat{\Omega}) - h_{ij}(t_e, \hat{\Omega})] \\
 z(t) &= \int_{S^2} d\hat{\Omega} z(t, \hat{\Omega})
 \end{aligned}$$

- Timing residual in frequency-domain

$$\tilde{r}(f, \hat{\Omega}) = \frac{1}{2\pi i f} \left( 1 - e^{-2\pi i f L (1 + \hat{\Omega} \cdot \hat{p})} \right) \times \sum_A h_A(f, \hat{\Omega}) F^A(\hat{\Omega})$$

- Antenna pattern

$$F^A(\hat{\Omega}) = e_{ij}^A(\hat{\Omega}) \frac{\hat{p}^i \hat{p}^j}{2(1 + \hat{\Omega} \cdot \hat{p})}$$

# Detecting a GWB with PTA

- Assume the GWB is isotropic, unpolarized, and stationary

$$\left\langle h_A^*(f, \hat{\Omega}) h_{A'}(f', \hat{\Omega}') \right\rangle = \frac{3H_0^2}{32\pi^3 f^3} \delta^2(\hat{\Omega}, \hat{\Omega}') \delta_{AA'} \delta(f - f') \Omega_{\text{gw}}(f)$$

- Spectrum of GWB

$$\Omega_{\text{gw}}(f) \equiv \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{d \ln f}, \quad \rho_{\text{crit}} = \frac{3H_0^2}{8\pi}, \quad \rho_{\text{gw}} = \frac{1}{32\pi} \left\langle \dot{h}_{ij}(t, \vec{x}) \dot{h}^{ij}(t, \vec{x}) \right\rangle,$$

- Cross-power spectral density

$$S_{IJ} = \left\langle \tilde{r}_I^*(f) \tilde{r}_J(f') \right\rangle = \frac{1}{\gamma} \frac{H_0^2}{16\pi^4 f^5} \delta(f - f') \Gamma_{IJ}(f, L_I, L_J, \xi) \Omega_{\text{gw}}(f)$$

- Overlap reduction function (ORF) is function of  $f, L_I, L_J, \xi$

$$\Gamma_{IJ} = \gamma \sum_A \int d\hat{\Omega} \left( e^{2\pi i f L_I (1 + \hat{\Omega} \cdot \hat{p}_I)} - 1 \right) \times \left( e^{-2\pi i f L_J (1 + \hat{\Omega} \cdot \hat{p}_J)} - 1 \right) F_I^A(\hat{\Omega}) F_J^A(\hat{\Omega})$$

- Hellings & Downs correlations for  $fL \gg 1$  (short-wavelength approximation)

$$\Gamma_{IJ} = \frac{3}{2} \left( \frac{1 - \cos \xi}{2} \right) \ln \frac{1 - \cos \xi}{2} - \frac{1 - \cos \xi}{8} + \frac{1}{2}$$

# Time of arrivals (TOAs)

$$\tau = \tau^{\text{TM}} + n = \tau^{\text{TM}} + \tau^{\text{RN}} + \tau^{\text{DM}} + \tau^{\text{WN}} + \tau^{\text{GW}}$$

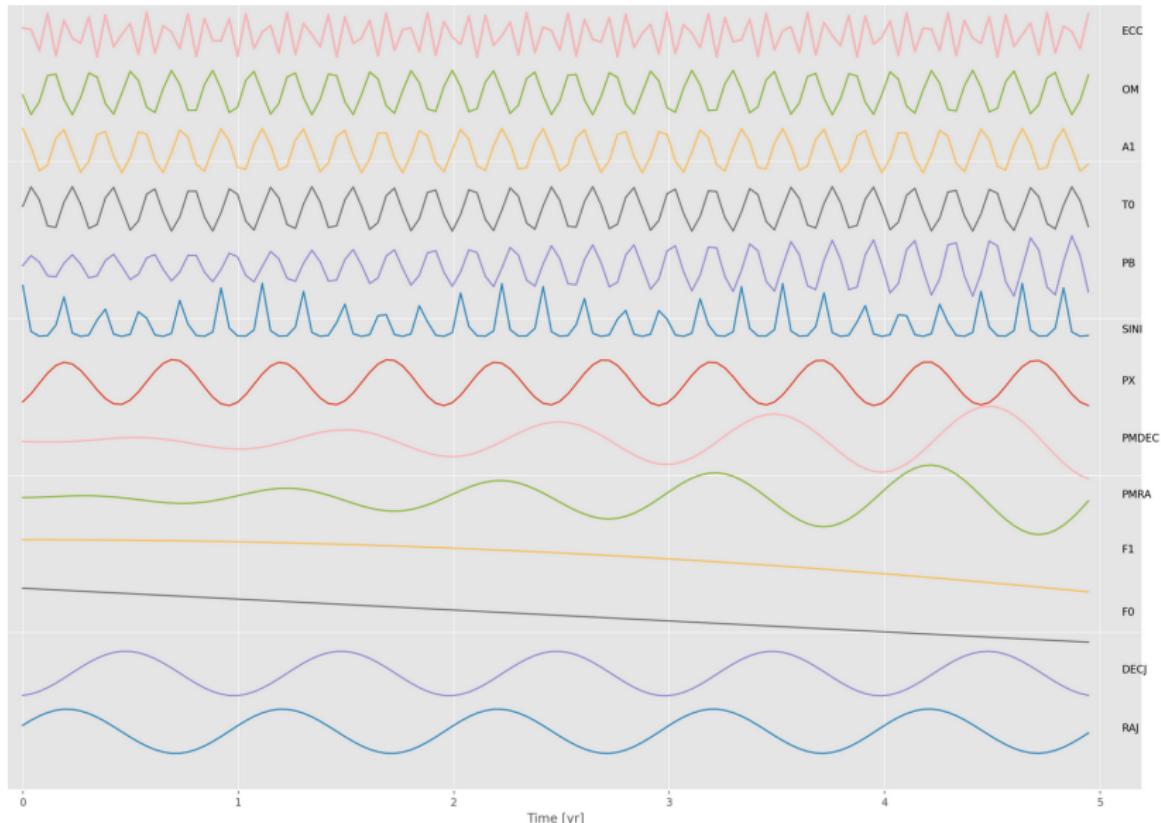
- $\tau^{\text{TM}}$  – timing model: physical model for TOAs taking in to account spin period, proper motion, binary orbital dynamics, etc.
- $\tau^{\text{RN}}$  – red noise (i.e. low-frequency correlated noise). Correlation timescales on the order of weeks - years
- $\tau^{\text{DM}}$ : Model for time-varying dispersion measure variations (i.e. has  $1/\nu^2$  dependence, where  $\nu$  is the radio frequency)
- $\tau^{\text{WN}}$  – white noise: it is more than just a variance since we have data taken from different observing systems and different telescopes.
- $\tau^{\text{GW}}$  – GW signal

## Timing residuals

$$\begin{aligned}
\delta\tau &= \tau^{\text{obs}} - \tau^{\text{det}}(\xi_{\text{est}}) \\
&= \tau^{\text{det}}(\xi_{\text{true}}) - \tau^{\text{det}}(\xi_{\text{est}}) + n \\
&= \tau^{\text{det}}(\xi_{\text{est}} + \epsilon) - \tau^{\text{det}}(\xi_{\text{est}}) + n \\
&= \tau^{\text{det}}(\xi_{\text{est}}) + \frac{\partial \tau^{\text{det}}(\xi_{\text{est}} + \epsilon)}{\partial \xi} \Big|_{\epsilon=0} \epsilon - \tau^{\text{det}}(\xi_{\text{est}}) + n + \mathcal{O}(\epsilon^2) \\
&\approx \frac{\partial \tau^{\text{det}}(\xi_{\text{est}} + \epsilon)}{\partial \xi} \Big|_{\epsilon=0} \epsilon + n \\
&= M\epsilon + n,
\end{aligned} \tag{1}$$

- $M$  is the design matrix and  $\epsilon$  is an offset parameter.
  - $\tau^{\text{TM}} \sim \text{milliseconds}$
  - $n \sim \text{nano- or microseconds}$

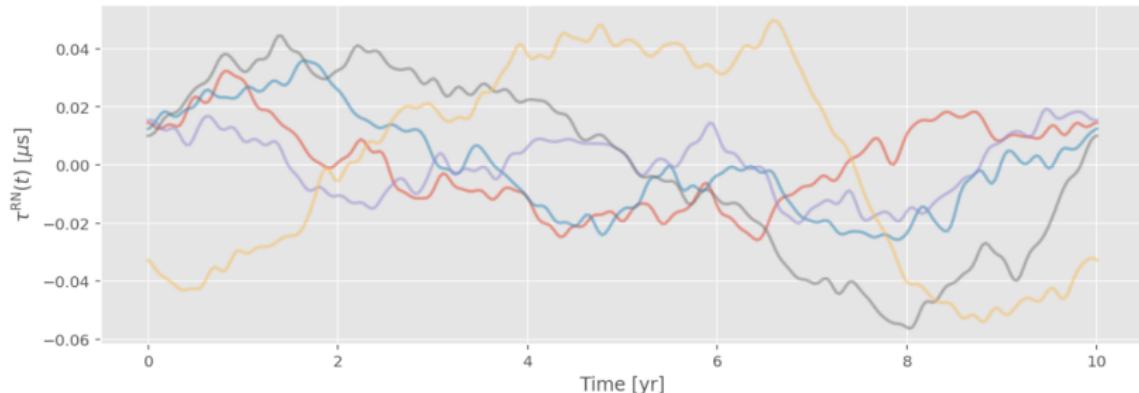
## Timing model



## Red noise

$$\tau_{\text{RN}} = \sum_{j=1}^{N_{\text{mode}}} \left[ a_j \sin\left(\frac{2\pi j t}{T}\right) + b_j \cos\left(\frac{2\pi j t}{T}\right) \right] = F_{\text{red}} a_{\text{red}},$$

- $a_{\text{red}}$  is a vector of the alternating sine and cosine amplitudes
  - $T$  is the total time span of the data
  - $F_{\text{red}}$  is a  $N_{\text{TOA}} \times 2N_{\text{mode}}$  matrix with alternating sine and cosine terms
  - $N_{\text{mode}}$  the number of frequencies used. Typically we use 50 Fourier modes.



# Red noise

- covariance matrix

$$\begin{aligned} K_{\text{red}} &= \langle \tau^{\text{RN}} (\tau^{\text{RN}})^T \rangle \\ &= F_{\text{red}} \langle a_{\text{red}} a_{\text{red}}^T \rangle F_{\text{red}}^T \\ &= F_{\text{red}} \varphi F_{\text{red}}^T \end{aligned}$$

- $\varphi = \langle a_{\text{red}} a_{\text{red}}^T \rangle$  is a matrix with zero off-diagonal elements

$$\varphi_{i,i} = P(f_i)$$

- Power spectrum

- power-law

$$P_{\text{PL}}(f; A, \gamma) = \frac{A^2}{12\pi^2} \left( \frac{f}{\text{yr}^{-1}} \right)^{-\gamma} \text{yr}^3$$

- broken power-law

$$P_{\text{BPL}}(f; A, \gamma, \delta, f_b, \kappa) = \frac{A^2}{12\pi^2} \left( \frac{f}{\text{yr}^{-1}} \right)^{-\gamma} \left( 1 + \left( \frac{f}{f_b} \right)^{1/\kappa} \right)^{\kappa(\gamma-\delta)} \text{yr}^3$$

- free spectrum

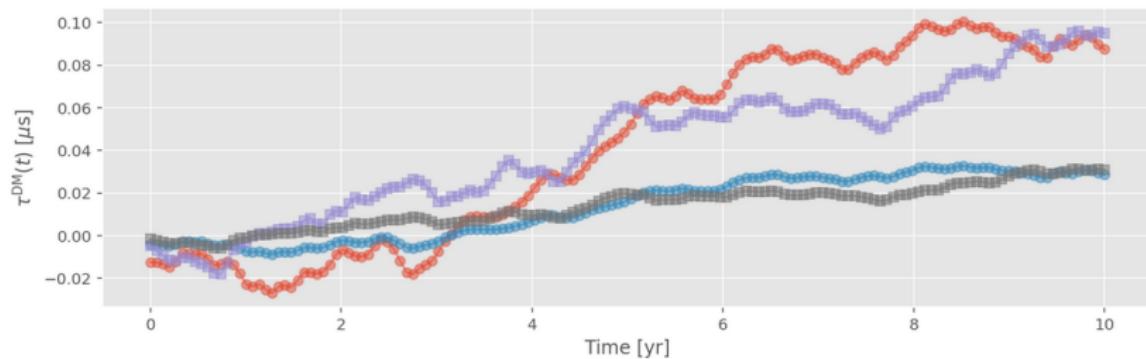
$$P_{\text{FS}}(f_i; \rho_i) = \rho_i^2 T,$$

$\rho_i$  is the spectral amplitude at frequency  $f_i = i/T$ .

# Dispersion measure variations

Dispersion measure is due to the propagation of radio waves through the charged plasma of the interstellar medium (ISM),

$$\text{DM}(t) = \int_0^{L(t)} n_e(\mathbf{x}) d\ell.$$



# Dispersion measure variations

- timing residual

$$\tau^{\text{DM}} = F_{\text{DM}} a_{\text{DM}}$$

- covariance matrix

$$\begin{aligned} K_{\text{DM}} &= F_{\text{DM}} \langle a_{\text{DM}} a_{\text{DM}}^T \rangle F_{\text{DM}}^T \\ &= F_{\text{DM}} \varphi_{\text{DM}} F_{\text{DM}}^T \end{aligned}$$

- $\varphi_{\text{DM}} = \langle a_{\text{DM}} a_{\text{DM}}^T \rangle$  is a matrix with zero off-diagonal elements

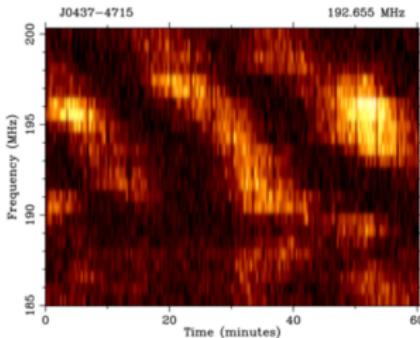
$$\varphi_{i,i} = P(f_i)$$

- radio frequency dependent power spectrum

$$P_{\text{DM}}(f; A_{\text{DM}}, \gamma_{\text{DM}}) = \frac{A_{\text{DM}}^2}{12\pi^2} f_{yr}^{-3} \left(\frac{f}{f_{yr}}\right)^{-\gamma_{\text{DM}}} \left(\frac{1400\text{MHz}}{\nu}\right)^2$$

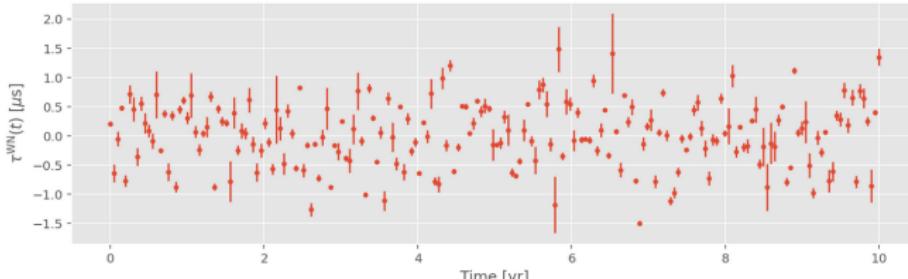
# White noise

- Measurement error is the biggest contributor.
- A function of radio frequencies and observation times.



- covariance matrix

$$N_{ij} = \delta_{ij} \left( \sigma_{\text{meas},ij}^2 + \sigma_{\text{equad},ij}^2 \right),$$



# GWB

$$\tau_{\text{GWB}} = \sum_{j=1}^{N_{\text{mode}}} \left[ a_j \sin\left(\frac{2\pi j t}{T}\right) + b_j \cos\left(\frac{2\pi j t}{T}\right) \right] = F_{\text{GWB}} a_{\text{GWB}},$$

- covariance matrix

$$\begin{aligned} K_{\text{GWB}} &= \langle \tau^{\text{GWB}} (\tau^{\text{GWB}})^T \rangle \\ &= F_{\text{GWB}} \langle a_{\text{GWB}} a_{\text{GWB}}^T \rangle F_{\text{GWB}}^T \\ &= F_{\text{GWB}} \varphi F_{\text{GWB}}^T \end{aligned}$$

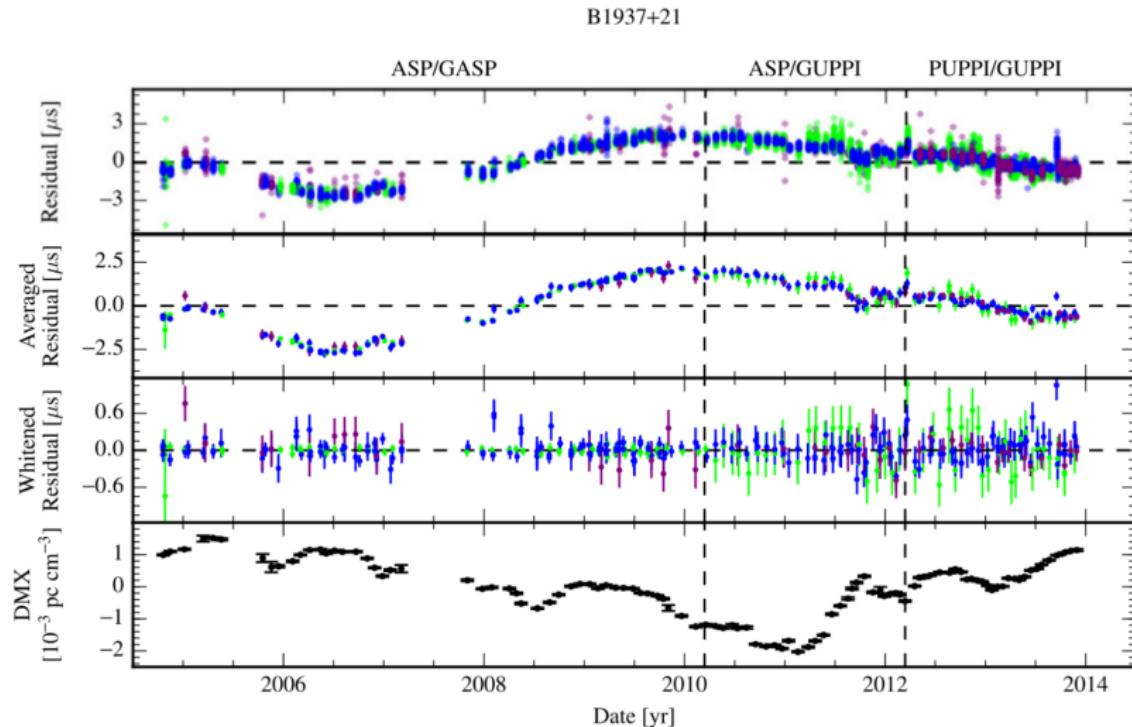
- correlations

$$\varphi_{IJ;i,i} = \Gamma_{I,J} P(f_i)$$

- Power spectrum from SMBHBs

$$P(f; A, \gamma) = \frac{A^2}{12\pi^2} \left( \frac{f}{\text{yr}^{-1}} \right)^{-\gamma} \text{yr}^3$$

# Putting them all together



- timing residuals

$$\begin{aligned}\delta\tau &= M\epsilon + F_{\text{red}}a_{\text{red}} + F_{\text{DM}}a_{\text{DM}} + n \\ &= Tb + n\end{aligned}$$

- definitions

$$T = [M \ F_{\text{red}} \ F_{\text{DM}}]; \quad b = \begin{bmatrix} \epsilon \\ a_{\text{red}} \\ a_{\text{DM}} \end{bmatrix} \quad B = \begin{bmatrix} \infty & & \\ & \varphi & \\ & & \varphi_{\text{DM}} \end{bmatrix}$$

- covariance matrix

$$C = N + K = N + TBT^T$$

where  $N = \langle nn^T \rangle$  is covariance matrix for white noise.

# likelihood

- Basis Picture:

$$p(\delta\tau|b, \phi) = \frac{\exp \left[ -\frac{1}{2}(\delta\tau - Tb)^T N^{-1}(\delta\tau - Tb) \right]}{\sqrt{\det 2\pi N}} \frac{\exp \left[ -\frac{1}{2}b^T B^{-1}b \right]}{\sqrt{\det 2\pi B}}$$

- Kernel Picture

$$p(\delta\tau|\phi) = \frac{\exp \left[ -\frac{1}{2}\delta\tau^T C^{-1}\delta\tau \right]}{\sqrt{\det 2\pi C}}$$

- Woodbury Lemma

$$C^{-1} = (N + TBT^T)^{-1} = N^{-1} - N^{-1}T \left( B^{-1} + T^T N^{-1} T \right)^{-1} T^T N^{-1}$$

$$\det C = \det (N + TBT^T) = \det (N) \det (B) \det (B^{-1} + T^T N^{-1} T)$$

$B \sim 1000 \times 1000$  and  $C \sim 30000 \times 30000$  means speedup of  $\sim 1000$

- Bayes' theorem

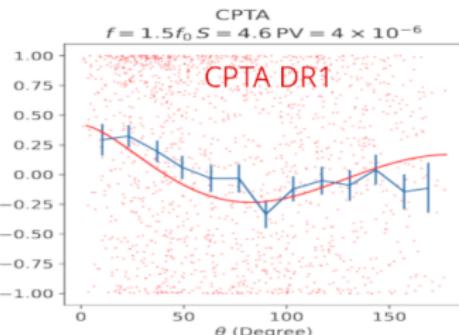
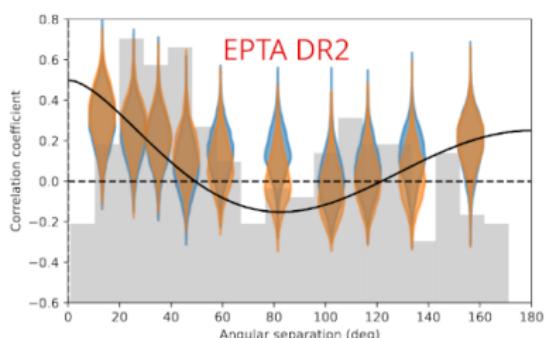
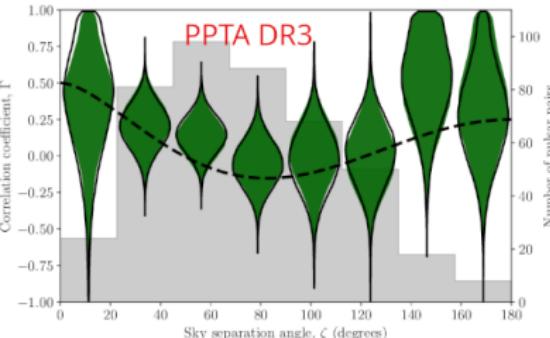
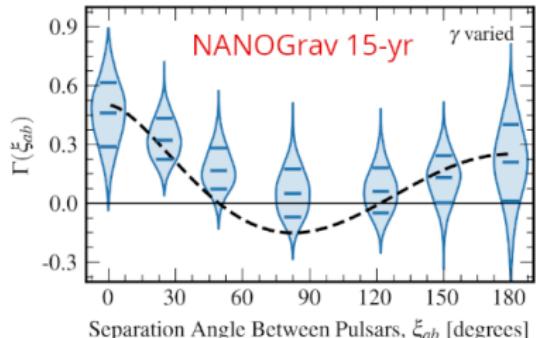
- Bayes factor

$$BF_{12} = \frac{\Pr(\mathcal{D} | \mathcal{H}_1)}{\Pr(\mathcal{D} | \mathcal{H}_2)}$$

**Table 7.1** Evidence categories for the Bayes factor  $BF_{12}$  (Jeffreys, 1961).

Bayes factor $BF_{12}$			Interpretation
	>	100	Extreme evidence for $\mathcal{M}_1$
30	–	100	Very strong evidence for $\mathcal{M}_1$
10	–	30	Strong evidence for $\mathcal{M}_1$
3	–	10	Moderate evidence for $\mathcal{M}_1$
1	–	3	Anecdotal evidence for $\mathcal{M}_1$
	1		No evidence
1/3	–	1	Anecdotal evidence for $\mathcal{M}_2$
1/10	–	1/3	Moderate evidence for $\mathcal{M}_2$
1/30	–	1/10	Strong evidence for $\mathcal{M}_2$
1/100	–	1/30	Very strong evidence for $\mathcal{M}_2$
	<	1/100	Extreme evidence for $\mathcal{M}_2$

# The stochastic signal in PTAs (2023-06-29)



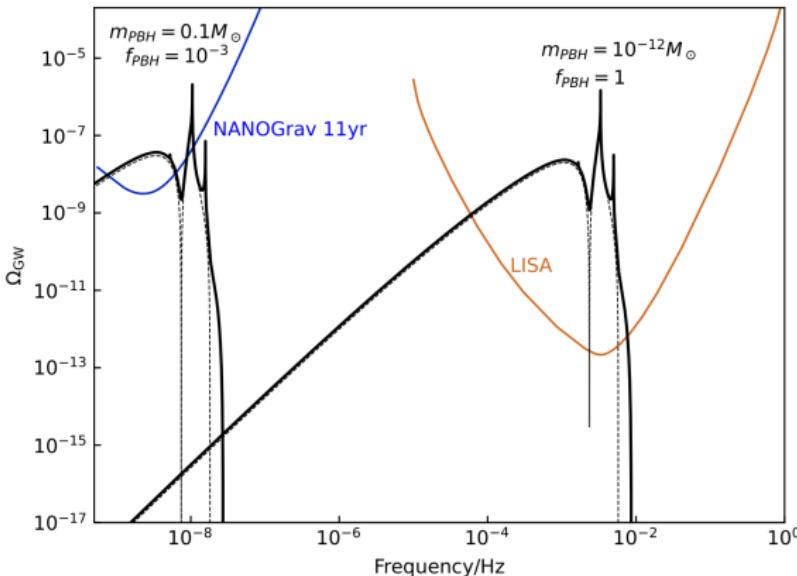
NANOGrav, 951 (2023) 1, L8; PPTA, ApJL 951 (2023) 1, L6

EPTA+InPTA, A&A 678 (2023) A50; CPTA, RAA 23 (2023) 7, 075024

# Detect Scalar-Induced GW (SIGW) with PTA

Primordial perturbations can be generated by quantum fluctuations during inflation

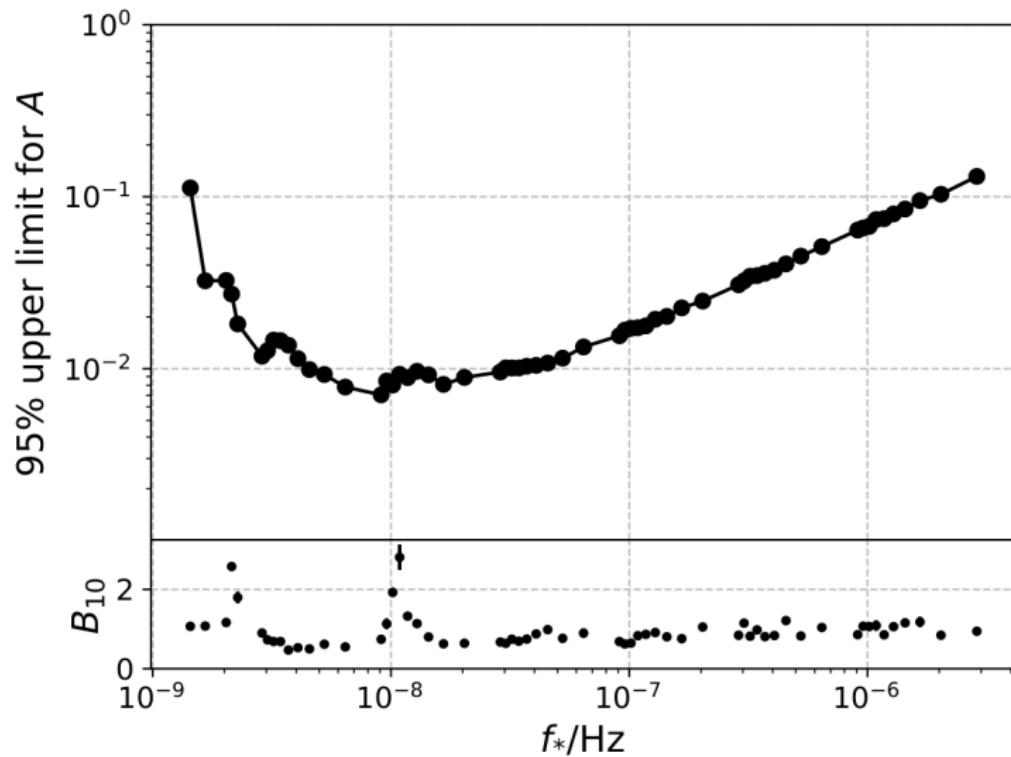
$$ds^2 = a^2 \left\{ -(1 + 2\phi)d\eta^2 + \left[ (1 - 2\phi)\delta_{ij} + \frac{h_{ij}}{2} \right] dx^i dx^j \right\} \quad (2)$$



Chen Yuan, ZCC, Qing-Guo Huang<sup>†</sup>, 1906.11549 (PRD Rapid Communications)

See also: [Rong-Gen Cai, Shi Pi, Shao-Jiang Wang, Xing-Yu Yang, 1907.06372 \( JCAP\)](#)

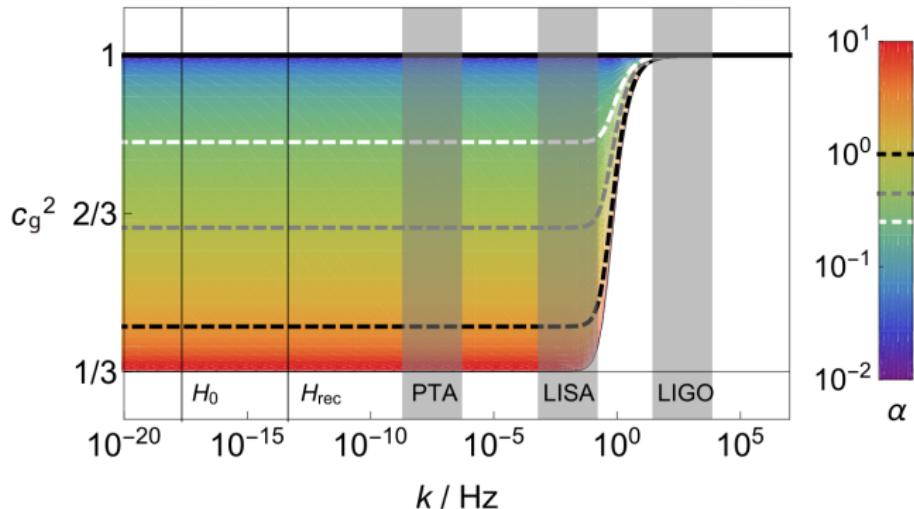
# Constrain SIGWs with NANOGrav 11-yr data set



ZCC, Chen Yuan, Qing-Guo Huang<sup>†</sup>, 1910.12239 (PRL)

## Speed of GW

- GW170817:  $-3 \times 10^{-15} \leq c_g - 1 \leq 7 \times 10^{-16}$   
*LVK, 1710.05832 (PRL)*
  - $c_g$  can be frequency dependent



*Claudia de Rham, Scott Melville, 1806.09417 (PRL)*

# Speed of SIGW

- EoM

$$h_{\mathbf{k}}''(\eta) + 2\mathcal{H}h_{\mathbf{k}}'(\eta) + \textcolor{red}{c_g^2}k^2h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta). \quad (3)$$

- SIGW spectrum

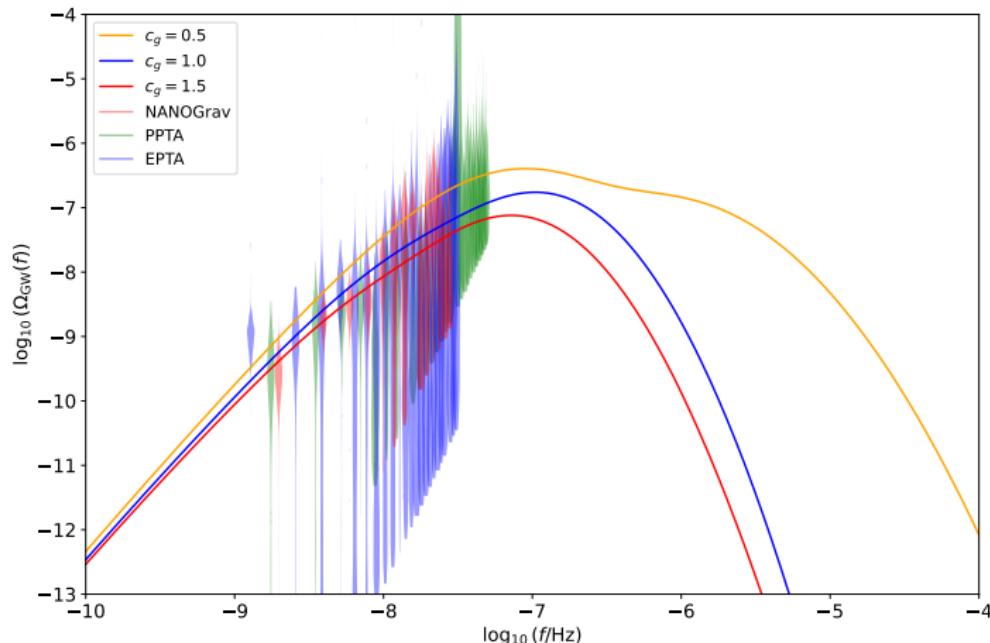
$$\Omega_{\text{GW}}(k) = \int_0^\infty dv \int_{|1-v|}^{1+v} du \mathcal{T}(u, v, \textcolor{red}{c_g}) P_\zeta(vk) P_\zeta(uk). \quad (4)$$

- Transfer function

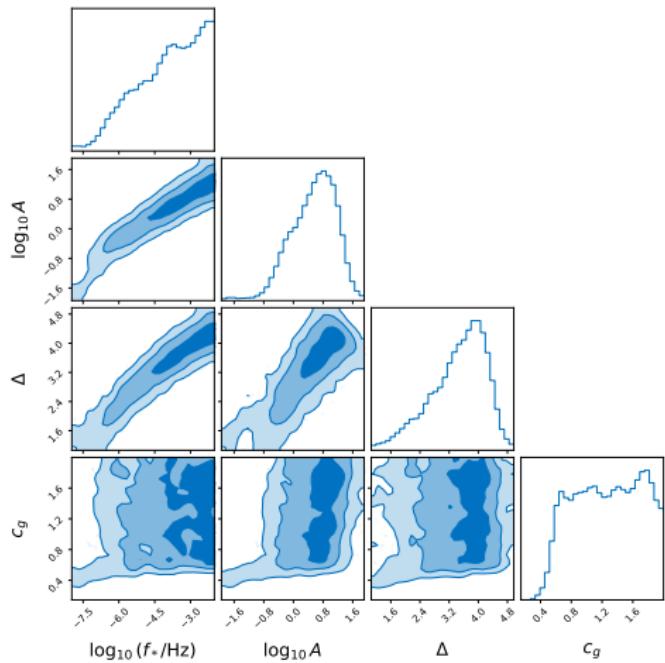
$$\begin{aligned} \mathcal{T}(u, v, \textcolor{red}{c_g}) &= \frac{3 \left[ 4v^2 - (v^2 - u^2 + 1)^2 \right]^2 (v^2 + u^2 - 3c_g^2)^2}{1024v^8u^8} \\ &\times \left\{ \left[ (v^2 + u^2 - 3c_g^2) \ln \left( \left| \frac{3c_g^2 - (v+u)^2}{3c_g^2 - (v-u)^2} \right| \right) - 4vu \right]^2 \right. \\ &\quad \left. + \pi^2 (v^2 + u^2 - 3c_g^2)^2 \Theta(v+u-\sqrt{3}c_g) \right\}. \end{aligned} \quad (5)$$

Jun Li, Guang-Hai Guo, 2312.04589

PE with NANOGrav 15-yr data set + PPTA DR3 + EPTA DR2



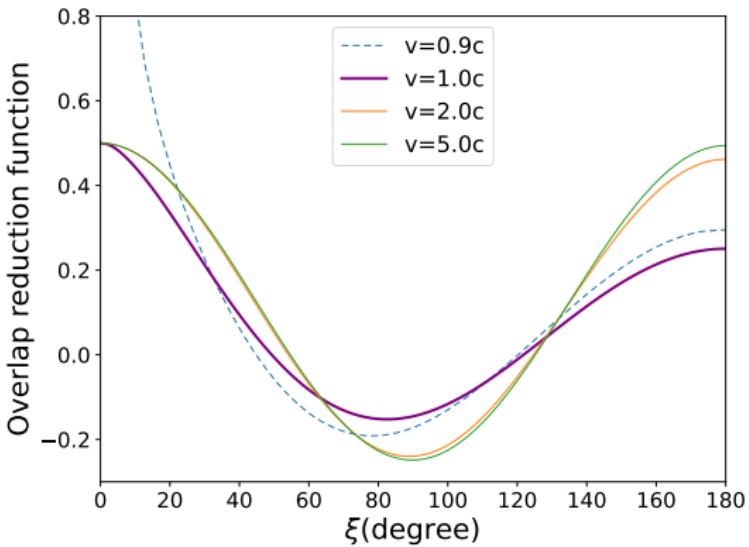
ZCC, Jun Li, Lang Liu<sup>†</sup>, Zhu Yi, 2401.09818 (PRDL)



- $c_g \gtrsim 0.61$  at a 95% CI.
  - Consistent with  $c_q = 1$ .

ZCC, Jun Li, Lang Liu<sup>†</sup>, Zhu Yi, 2401.09818 (PRDL)

## ORF

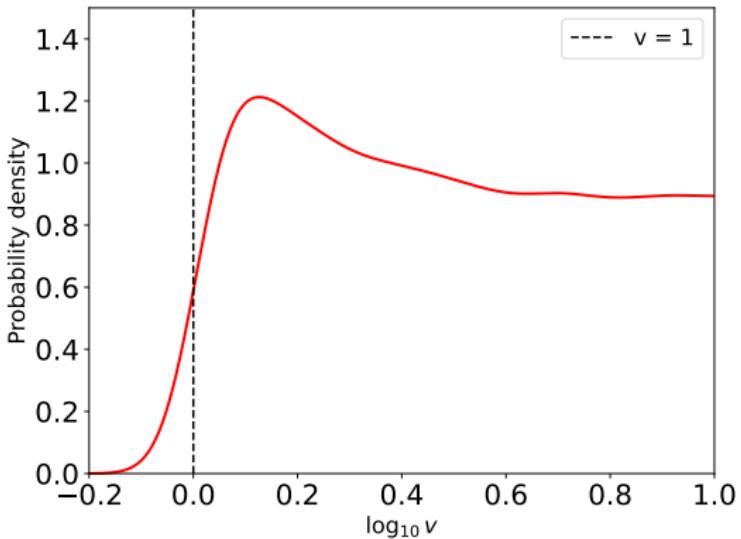


Reginald Christian Bernardo, Kin-Wang Ng, 2208.12538, (PRD)

Reginald Christian Bernardo, Kin-Wang Ng, 2302.11796, (PRDL)

Yan-Chen Bi, Yu-Mei Wu<sup>†</sup>, ZCC<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.08366 (PRDL)

## PE with NANOGrav 15-yr data set

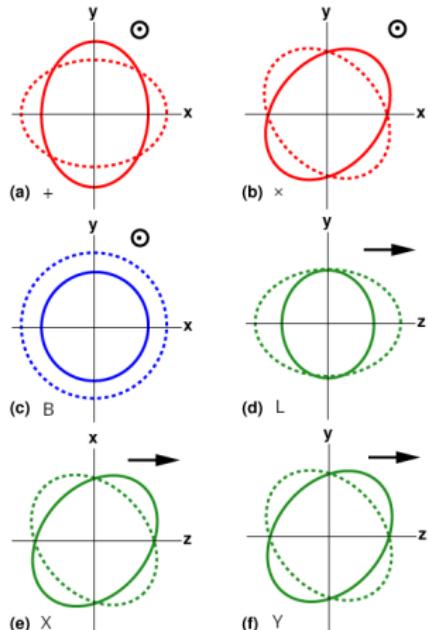


- $c_g \gtrsim 0.85$ .
  - Still consistent with  $c_g = 1$ .

Yan-Chen Bi, Yu-Mei Wu<sup>†</sup>, ZCC<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.08366 (PRDL)

# Alternative Polarizations

## Gravitational-Wave Polarization



- A general metric gravity theory in 4D spacetime can have 6 polarization modes.
- polarization tensors

$$\epsilon_{ij}^+ = \hat{m} \otimes \hat{m} - \hat{n} \otimes \hat{n},$$

$$\epsilon_{ij}^\times = \hat{m} \otimes \hat{n} + \hat{n} \otimes \hat{m},$$

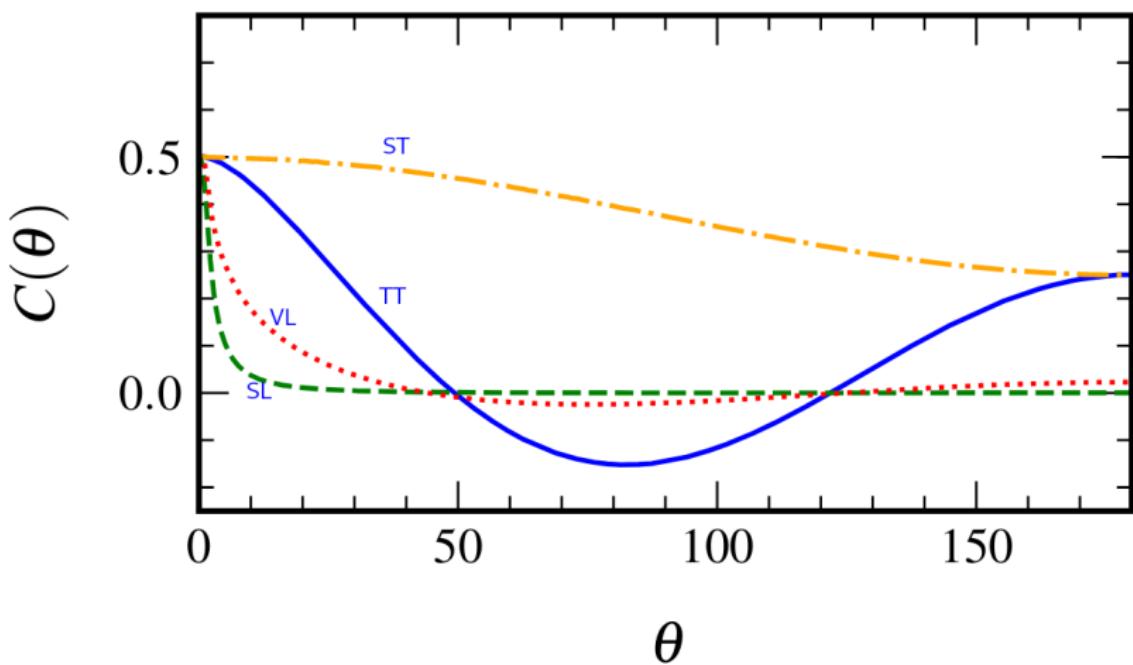
$$\epsilon_{ij}^B = \hat{m} \otimes \hat{m} + \hat{n} \otimes \hat{n},$$

$$\epsilon_{ij}^L = \hat{\Omega} \otimes \hat{\Omega},$$

$$\epsilon_{ij}^X = \hat{m} \otimes \hat{\Omega} + \hat{\Omega} \otimes \hat{m},$$

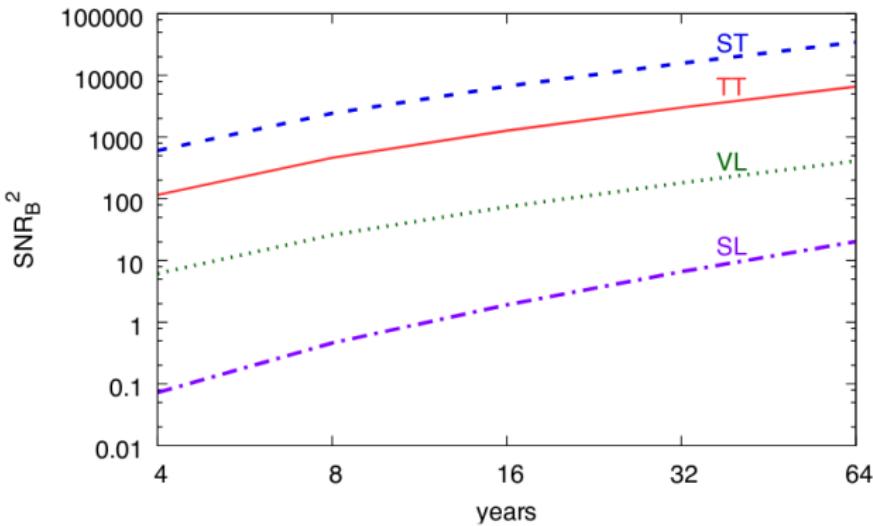
$$\epsilon_{ij}^Y = \hat{n} \otimes \hat{\Omega} + \hat{\Omega} \otimes \hat{n}$$

## ORF



$$|\Gamma_{ST}| > |\Gamma_{TT}| > |\Gamma_{VL}| > |\Gamma_{SL}|$$

$$\text{SNR}_B^2 = 2 \sum_f \sum_a^{N_p} \sum_{b>a}^{N_p} \frac{\Gamma_{ab}^I(f)}{\Gamma_{aa}^I(f)\Gamma_{bb}^I(f) + \Gamma_{ab}^I(f)}.$$



ST is the easiest to detect among the four polarization modes.

Neil J. Cornish, Logan O'Beirne, Stephen R. Taylor, Nicolás Yunes, 1712.07132 (PRL)

# Evidence for the ST correlations in NANOGrav 12.5-yr

- [ZCC, Chen Yuan, Qing-Guo Huang<sup>†</sup>, 2101.06869 \(SCPMA\)](#)

	TT	ST	VL	SL	ST+TT
DE438	4.96(9)	107(7)	1.94(3)	0.373(5)	96(3)

- Our results were reproduced  $\sim 8$  months later by [NANOGrav, 2109.14706 \(ApJL\)](#)

As shown in Fig. 10, the most favored Bayesian model is a GWB with GW-like monopolar correlations of Eq. (25) with a Bayes factor greater than 100. Additionally, as a cross-check, we have reproduced the results of Chen et al. (2021), where a model with ST correlations with a spectral index of 5, [ST]M3A[5], was compared to a model without correlations and a spectral index of 13/3, M2A[13/3]. We obtain a Bayes factor of

# Search for alternative polarizations in NANOGrav 15-yr data set

- Our paper appeared on arXiv one day prior to NANOGrav's. Both sets of results are broadly consistent with each other.
- [ZCC, Yu-Mei Wu<sup>†</sup>, Yan-Chen Bi<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.11238 \(PRD\)](#)

Model	ST	VL	SL	GTb	TT + ST
BF	0.40(3)	0.12(2)	0.002(1)	3.9(3)	0.943(5)

- [NANOGrav, 2310.12138 \(ApJL\)](#)

Our Bayesian analyses show the Bayes factor for HD over ST is  $\sim 2$ , and the Bayes factor for a model with both correlations compared to a model with just HD is  $\sim 1$ . These results are largely consistent with a similar study by Chen et al. (2023), in which they searched NANOGrav's 15 yr data set for nontensorial GWBs on a similar timescale to the work presented here. Taking the spectral parameter recovery into account, as in Figure 3, we found each correlation, when fit for individually, is in agreement with CURN. We also found more informative  $\log_{10} A_g$  and  $\gamma_g$  recovery for HD than ST, and HD parameters show better agreement with CURN spectral parameters when correlations are included together. The analyses in this Letter, as well as those in Bernardo & Ng (2023c) and Chen et al. (2023), do not rule out the possibility of ST correlations in our data. However, our analysis also shows no statistical need for an additional stochastic process with ST correlations.

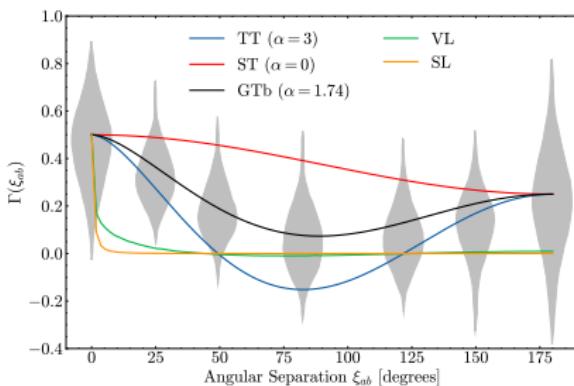
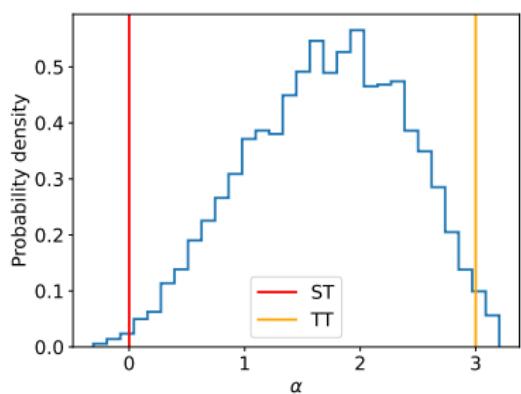
We also consider a parameterized transverse ORF as

$$\Gamma_{ab}(f) = \frac{1}{8} (3 + 4\delta_{ab} + \cos \xi_{ab}) + \frac{\alpha}{2} k_{ab} \ln k_{ab}. \quad (6)$$

ST:  $\alpha = 0$

TT:  $\alpha = 3$

prior of  $\alpha$ : Uniform(-10, 10)



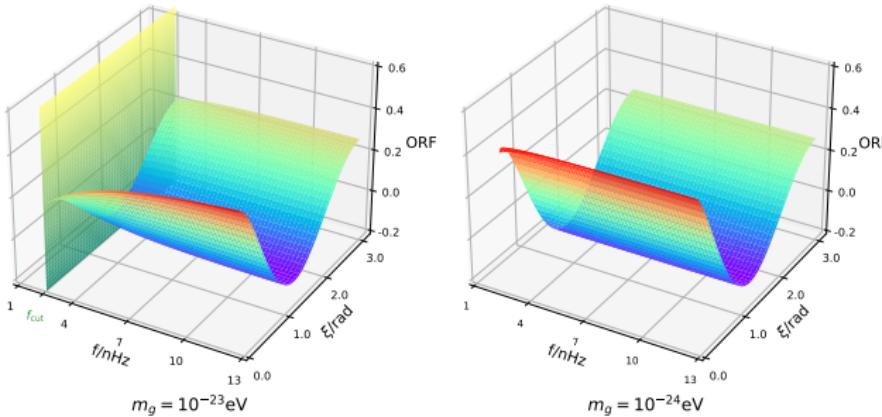
- Our analysis yields  $\alpha = 1.74^{+1.18}_{-1.41}$ , thus excluding both the TT and ST models at the 90% CL.

ZCC, Yu-Mei Wu<sup>†</sup>, Yan-Chen Bi<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.11238 (PRD)

# Massive Gravity



- Minimum frequency

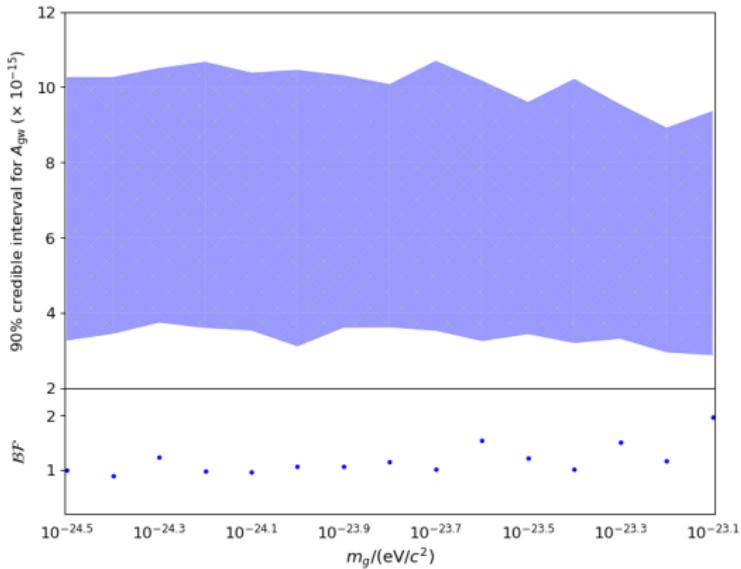


Qiuyue Liang, Mark Trodden, 2108.05344 (PRD)

Yu-Mei Wu, ZCC<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2302.00229 (PRD)

Yu-Mei Wu, ZCC<sup>†</sup>, Yan-Chen Bi<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.07469 (CQG)

## Constrain massive gravity with NANOGrav 15-yr data set



- No statistical evidence for massive graviton.
  - From  $f_{\min} < 1/T$ , where  $T = 16.03$  yr, we get

$$m_g < 8.2 \times 10^{-24} \text{ eV/c}^2. \quad (9)$$

Yu-Mei Wu, ZCC<sup>†</sup>, Yan-Chen Bi<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.07469 (CQG)

See also: *Sai Wang, Zhi-Chao Zhao, 2307.04680, PRDL*

# Summary

- PTAs are promising tools for probing modified gravity theories, including:
  - Speed of GW  
*Yan-Chen Bi, Yu-Mei Wu<sup>†</sup>, ZCC<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.08366 (PRDL)*  
*ZCC, Jun Li, Lang Liu<sup>†</sup>, Zhu Yi, 2401.09818 (PRDL)*
  - Alternative polarizations  
*ZCC, Chen Yuan, Qing-Guo Huang<sup>†</sup>, 2101.06869 (SCPMA)*  
*Yu-Mei Wu, ZCC<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2108.10518 (ApJ)*  
*ZCC, Yu-Mei Wu, Qing-Guo Huang<sup>†</sup>, 2109.00296 (CTP)*  
*ZCC, Yu-Mei Wu<sup>†</sup>, Yan-Chen Bi<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.11238 (PRD)*
  - Massive gravity  
*Yu-Mei Wu, ZCC<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2302.00229 (PRD)*  
*Yu-Mei Wu, ZCC<sup>†</sup>, Yan-Chen Bi<sup>†</sup>, Qing-Guo Huang<sup>†</sup>, 2310.07469 (CQG)*
- See Lang Liu (柳浪)'s talk on probing new physics with PTAs.
- The search for new physics using the IPTA DR3 is underway ...