Probing Scalar-induced Gravitational Waves through Pulsar Timing Arrays

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Based on *2307.01102, 2307.14911*

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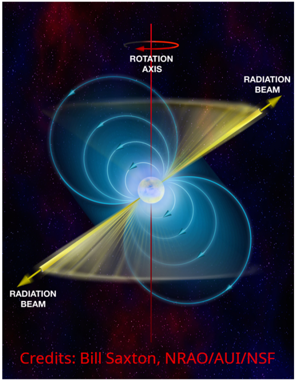
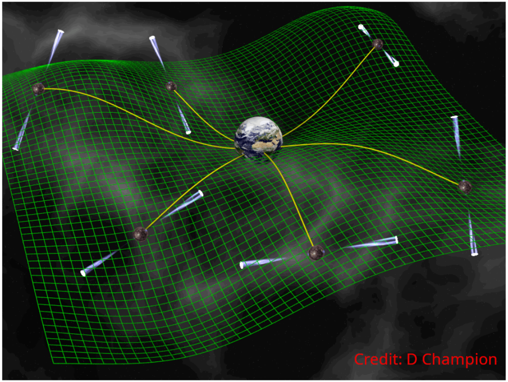
Outline

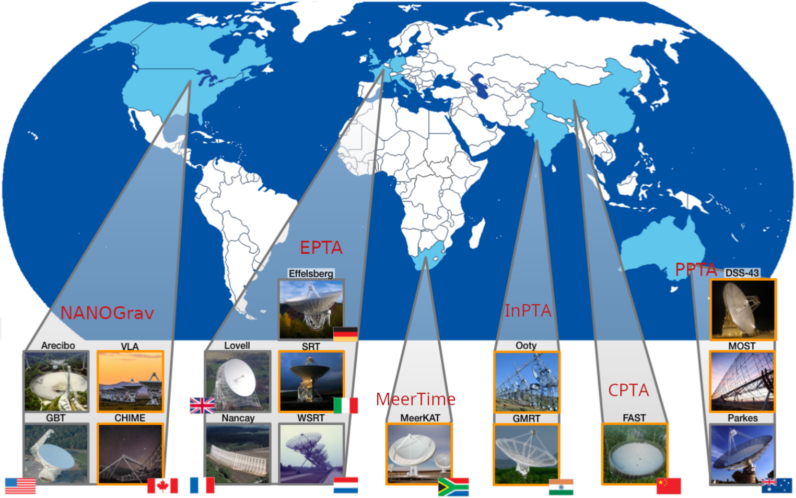
Pulsar and PTA





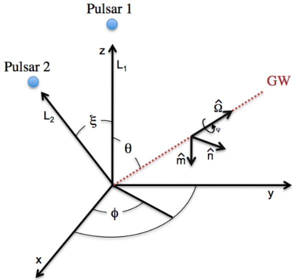
- Pulsars are highly magnetized, rotating neutron stars that emit regular pulses of electromagnetic radiation.
- GWs can cause tiny distortion in spacetime inducing variations in the time of arrivals (ToAs).
- A pulsar timing array (PTA) pursues to detect nHz GWs by regularly monitoring ToAs from an array of the ultra rotational stable millisecond pulsars.

PTAs in operation



IPTA: PPTA + EPTA + NANOGrav + InPTA
Observers: CPTA, MeerTime

Timing residual induced by a GWB



- Redshift

$$z(t,\hat{\Omega}) = \frac{\nu_e - \nu_p}{\nu_p}$$
$$= \frac{\hat{p}^i \hat{p}^j}{2(1+\hat{\Omega}\cdot\hat{p})}\left[h_{ij}\left(t_p,\hat{\Omega}\right) - h_{ij}\left(t_e,\hat{\Omega}\right)\right]$$
$$z(t) = \int_{S^2} d\hat{\Omega}\, z(t,\hat{\Omega})$$

- Timing residual in frequency-domain

$$\tilde{r}(f,\hat{\Omega}) = \frac{1}{2\pi i f}\left(1 - e^{-2\pi i f L(1+\hat{\Omega}\cdot\hat{p})}\right) \times \sum_A h_A(f,\hat{\Omega}) F^A(\hat{\Omega})$$

- Antenna pattern

$$F^A(\hat{\Omega}) = e^A_{ij}(\hat{\Omega}) \frac{\hat{p}^i \hat{p}^j}{2(1+\hat{\Omega}\cdot\hat{p})}$$

- Assume the GWB is isotropic, unpolarized, and stationary

$$\left\langle h_A^*(f,\hat{\Omega}) h_{A'}(f',\hat{\Omega}') \right\rangle = \frac{3H_0^2}{32\pi^3 f^3}\delta^2(\hat{\Omega},\hat{\Omega}')\delta_{AA'}\delta(f-f')\Omega_{\rm gw}(f)$$

- Spectrum of GWB

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_{\rm crit}}\frac{d\rho_{\rm gw}}{d\ln f}, \qquad \rho_{\rm crit} = \frac{3H_0^2}{8\pi}, \quad \rho_{\rm gw} = \frac{1}{32\pi}\left\langle \dot{h}_{ij}(t,\vec{x})\dot{h}^{ij}(t,\vec{x}) \right\rangle,$$

- Cross-power spectral density

$$\boxed{S_{IJ} = \langle \tilde{r}_I^*(f)\tilde{r}_J(f')\rangle = \frac{1}{\gamma}\frac{H_0^2}{16\pi^4 f^5}\delta\left(f-f'\right)\Gamma_{IJ}(f,L_I,L_J,\xi)\,\Omega_{\rm gw}(f)}$$

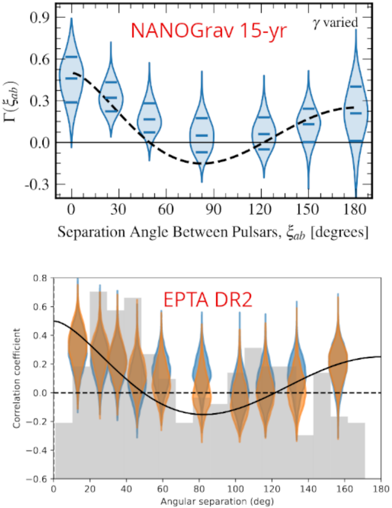
- Overlap reduction function (ORF) is function of f, L_I, L_J, ξ

$$\boxed{\Gamma_{IJ} = \gamma\sum_A\int d\hat{\Omega}\left(e^{2\pi i f L_I\left(1+\hat{\Omega}\cdot\hat{p}_I\right)}-1\right)\times\left(e^{-2\pi i f L_J\left(1+\hat{\Omega}\cdot\hat{p}_J\right)}-1\right)F_I^A(\hat{\Omega})F_J^A(\hat{\Omega})}$$

- Hellings & Downs correlations for $fL \gg 1$ (short-wavelength approximation)

$$\boxed{\Gamma_{IJ} = \frac{3}{2}\left(\frac{1-\cos\xi}{2}\right)\ln\frac{1-\cos\xi}{2} - \frac{1-\cos\xi}{8} + \frac{1}{2}}$$

The stochastic signal in PTAs



Gabriella Agazie, et al., ApJL (2023); Daniel Reardon, et al. ApJL (2023)

J. Antoniadis, et al., A&A (2023); Heng Xu, et al., RAA (2023)

SIGWs can explain the PTA signal.

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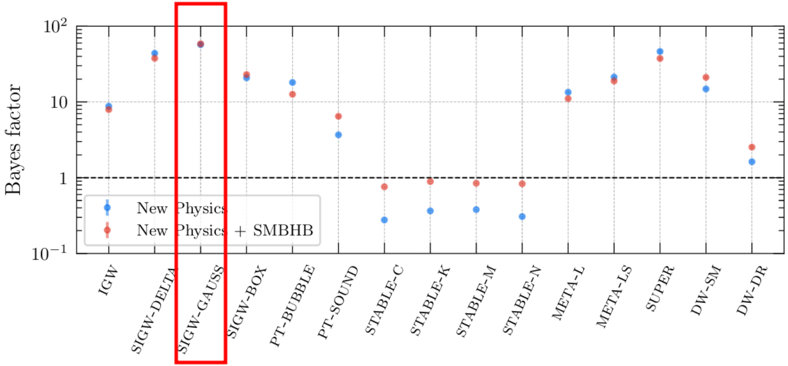


Figure 2. Bayes factors for the model comparisons between the new-physics interpretations of the signal considered in this work and the interpretation in terms of SMBHBs alone. Blue points are for the new physics alone, and red points are for the new physics in combination with the SMBHB signal. We also plot the error bars of all Bayes factors, which we obtain following the bootstrapping method outlined in Section 3.2. In most cases, however, these error bars are small and not visible.

Adeela Afzal, ApJL (2023)

see also *Zhi-Qiang You, Zhu Yi, You Wu, 2307.04419*

Contributions from BNUZ

1. Zhi-Qiang You, Zhu Yi, You Wu, 2307.04419, JCAP under review
2. Zhi-Qiang You, Zhu Yi, You Wu, 2308.05632, JCAP under review
3. Jia-Heng Jin, Zu-Cheng Chen, Zhu Yi, Zhi-Qiang You, Lang Liu, You Wu, 2307.08687, JCAP under review
4. Zhu Yi, Qing Gao, Yungui Gong, Yue Wang, Fengge Zhang, 2307.02467, PRDL under review
5. Yu-Mei Wu, Zu-Cheng Chen, Qing-Guo Huang, 2307.03141, submitted to PRL
6. Yan-Chen Bi, Yu-Mei Wu, Zu-Cheng Chen, Qing-Guo Huang, 2307.00722, PRL under review
7. Lang Liu, Zu-Cheng Chen, Qing-Guo Huang, 2307.01102, PRL under review
8. Lang Liu, Zu-Cheng Chen, Qing-Guo Huang, 2307.14911, PRL under review

Scalar-Induced Gravitational Waves (SIGWs)

- Primordial perturbations can be generated by quantum fluctuations during inflation.

- Metric perturbation in Newtonian gauge

$$\mathrm{d}s^2 = a^2 \left\{ -(1+2\phi)\mathrm{d}\eta^2 + \left[(1-2\phi)\delta_{ij} + \frac{h_{ij}}{2} \right] \mathrm{d}x^i \mathrm{d}x^j \right\}, \tag{1}$$

where $\phi \equiv \phi^{(1)}$ and $h_{ij} \equiv h_{ij}^{(2)}$ are the scalar and tensor perturbations, respectively.

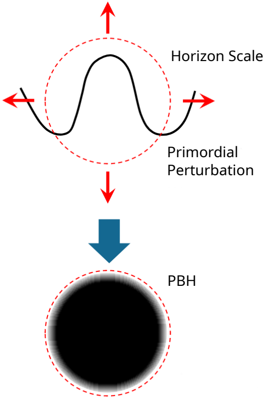
- Primordial scalar perturbation can be the source of SIGWs, as well as primordial black holes (PBHs).

Primordial black holes (PBHs)

- PBHs are formed in the early universe by gravitational collapse of primordial density perturbations
- PBH mass can span many orders

$$m_{\rm PBH} \sim \frac{t}{G} \sim 10^{-18} \left(\frac{t}{10^{-23}} \right) M_{\odot} \qquad (2)$$

- PBHs survived from Hawking radiation can be DM candidates.
- PBHs can explain LVK BBHs.



SIGW up to 3rd order

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\mathcal{T}_{ij}^{\ell m} S_{\ell m}(\phi)$$

The source term $S_{\ell m}(\phi)$ needs to be expanded to 4th order!

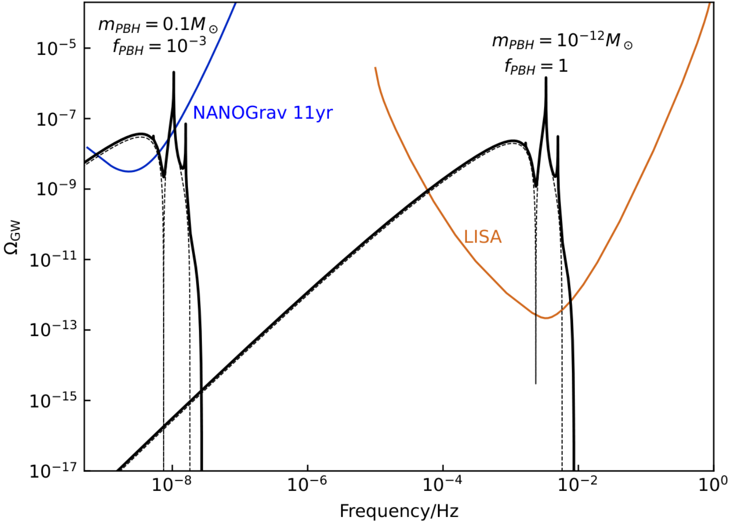
$$\Omega_{\mathrm{GW}}(\eta, k) = \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{d\ln f} \propto \left\langle S^{(2)} S^{(2)} \right\rangle + \left\langle S^{(3)} S^{(3)} \right\rangle + \left\langle S^{(2)} S^{(4)} \right\rangle$$

$$S_{ij}^{(2)} = 4\phi\partial_i\partial_j\phi + 2\partial_i\phi\partial_j\phi - \partial_i\left(\phi + \frac{\phi'}{\mathscr{H}}\right)\partial_j\left(\phi + \frac{\phi'}{\mathscr{H}}\right)$$

$$\begin{aligned} S_{ij}^{(3)} = &\frac{1}{\mathscr{H}}\left(12\mathscr{H}\phi - \phi'\right)\partial_i\phi\partial_j\phi - \frac{1}{\mathscr{H}^3}\left(4\mathscr{H}\phi - \phi'\right)\partial_i\phi'\partial_j\phi' \\ &+ \frac{1}{3\mathscr{H}^4}\left(2\partial^2\phi - 9\mathscr{H}\phi'\right)\partial_i\left(\mathscr{H}\phi + \phi'\right)\partial_j\left(\mathscr{H}\phi + \phi'\right) \end{aligned}$$

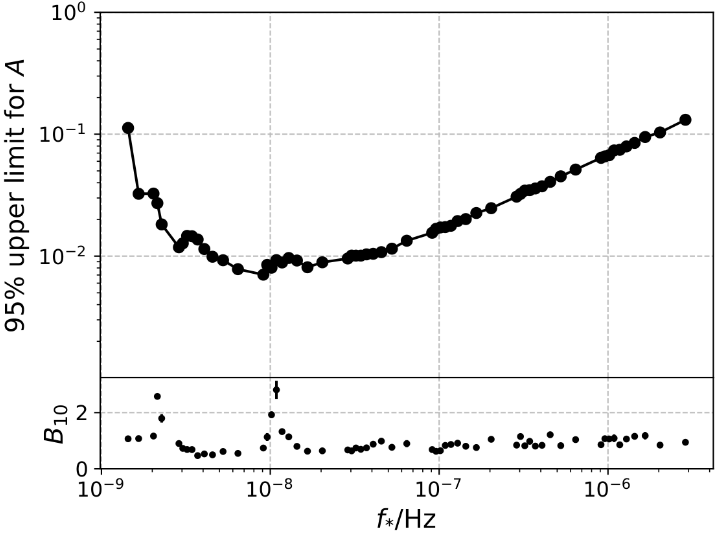
$$\begin{aligned} S_{ij}^{(4)} = &16\phi^3\partial_i\partial_j\phi + \frac{1}{3\mathscr{H}^3}\Big[2\phi'\partial^2\phi - 9\mathscr{H}\phi'^2 - 8\mathscr{H}\phi\partial^2\phi + 18\mathscr{H}^2\phi\phi' + 96\mathscr{H}^3\phi^2\Big]\partial_i\phi\partial_j\phi \\ &+ \frac{2}{3\mathscr{H}^5}\Big[-\phi'\partial^2\phi + 3\mathscr{H}\phi'^2 + 4\mathscr{H}\phi\partial^2\phi + 3\mathscr{H}^2\phi\phi' - 12\mathscr{H}^3\phi^2\Big]\partial_i\phi'\partial_j\phi' \\ &+ \frac{1}{36\mathscr{H}^6}\Big[-16(\partial^2\phi)^2 - 3\partial_k\phi'\partial^k\phi' + 120\mathscr{H}\phi'\partial^2\phi - 6\mathscr{H}\partial_k\phi\partial^k\phi' \\ &\qquad + 144\mathscr{H}^2\phi\partial^2\phi - 180\mathscr{H}^2\phi'^2 + 33\mathscr{H}^2\partial_k\phi\partial^k\phi - 504\mathscr{H}^3\phi\phi' - 144\mathscr{H}^4\phi^2\Big] \\ &\qquad \times \partial_i\left(\mathscr{H}\phi + \phi'\right)\partial_j\left(\mathscr{H}\phi + \phi'\right) \end{aligned}$$

Chen Yuan, ZCC, Qing-Guo Huang, PRD Rapid Communications (2019)



Chen Yuan, ZCC, Qing-Guo Huang, PRD Rapid Communications (2019)

Constrain SIGWs with NANOGrav 11-yr data set



ZCC, Chen Yuan, Qing-Guo Huang, PRL (2020)

Non-Gaussianity

- The local-type non-Gaussian curvature perturbations:

$$\mathcal{R}(\vec{x}) = \mathcal{R}_{\mathrm{G}}(\vec{x}) + F_{\mathrm{NL}}\left(\mathcal{R}_{\mathrm{G}}^2(\vec{x}) - \langle\mathcal{R}_{\mathrm{G}}^2(\vec{x})\rangle\right). \tag{3}$$

- The effective curvature power spectrum

$$P_{\mathcal{R}}^{\mathrm{NG}} = P_{\mathcal{R}}(k) + F_{\mathrm{NL}}^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u\, \frac{P_{\mathcal{R}}(uk) P_{\mathcal{R}}(vk)}{2u^2v^2}. \tag{4}$$

- The energy density of GWs

$$\Omega_{\mathrm{GW}}(k) = \int_0^\infty \mathrm{d}v \int_{|1-v|}^{|1+v|} \mathrm{d}u \mathcal{T}\, P_{\mathcal{R}}^{\mathrm{NG}}(vk) P_{\mathcal{R}}^{\mathrm{NG}}(uk), \tag{5}$$

where the transfer function $\mathcal{T} = \mathcal{T}(u, v)$ is given by

$$\begin{aligned}\mathcal{T}(u,v) =& \frac{3}{1024 v^8 u^8}\left[4v^2 - \left(v^2 - u^2 + 1\right)^2\right]^2 \left(v^2 + u^2 - 3\right)^2 \\ & \times \left\{\left[\left(v^2 + u^2 - 3\right)\ln\left(\left|\frac{3-(v+u)^2}{3-(v-u)^2}\right|\right) - 4vu\right]^2\right. \\ & \left. + \pi^2\left(v^2 + u^2 - 3\right)^2 \Theta(v + u - \sqrt{3})\right\}.\end{aligned} \tag{6}$$

Non-Gaussianity

- Power spectrum

$$P_{\mathcal{R}}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2}\right). \tag{7}$$

- The PBH mass fraction at formation time

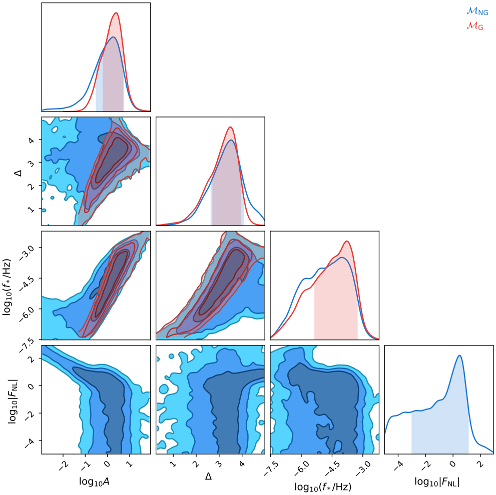
$$\beta(M) \simeq \frac{1}{2} \begin{cases} \operatorname{erfc}\left(\frac{\mathcal{R}_{\mathrm{G}}^{+}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right) + \operatorname{erfc}\left(-\frac{\mathcal{R}_{\mathrm{G}}^{-}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right); & F_{\mathrm{NL}} > 0, \\ \operatorname{erf}\left(\frac{\mathcal{R}_{\mathrm{G}}^{+}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right) - \operatorname{erf}\left(\frac{\mathcal{R}_{\mathrm{G}}^{-}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right); & F_{\mathrm{NL}} < 0, \end{cases} \tag{8}$$

with

$$\mathcal{R}_{\mathrm{G}}^{\pm}(\mathcal{R}) = \frac{1}{2F_{\mathsf{NL}}}\left(-1 \pm \sqrt{1 + 4F_{\mathsf{NL}}\mathcal{R} + 4F_{\mathsf{NL}}^2\langle\mathcal{R}_{\mathrm{G}}^2\rangle}\right). \tag{9}$$

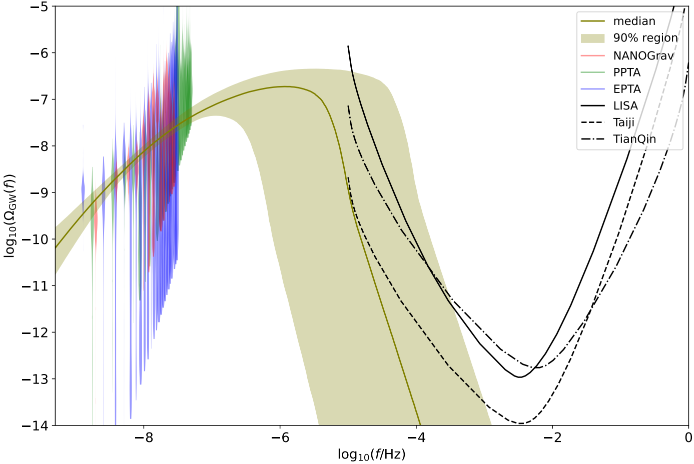
- The total abundance of PBHs in the dark matter at present

$$f_{\mathrm{PBH}} \equiv \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{CDM}}} = 2.7 \times 10^8 \int_{-\infty}^{\infty} \mathrm{d}\ln M \times \left(\frac{M}{M_\odot}\right)^{-1/2} \beta(M). \tag{10}$$



- $|F_{\rm NL}| \lesssim 13.9$
- $-13.9 \lesssim F_{\rm NL} \lesssim -0.1$ when further requiring $f_{\rm PBH} \lesssim 1$.

Lang Liu, ZCC, Qing-Guo Huang, 2307.01102



Lang Liu, ZCC, Qing-Guo Huang, 2307.01102

Implications

- The constraints on $F_{\rm NL}$ have significant implications for Multi-field inflation models.
- For instance, adiabatic curvaton models predict that

$$f_{\rm NL} = \frac{5}{3} F_{\rm NL} = \frac{5}{4 r_{\rm D}} - \frac{5 r_{\rm D}}{6} - \frac{5}{3}, \qquad (11)$$

where $r_{\rm D} = 3\rho_{\rm curvaton}/(3\rho_{\rm curvaton} + 4\rho_{\rm radiation})$ represents the "curvaton decay fraction" at the time of curvaton decay.

- Our constraint $|F_{\rm NL}| \lesssim 13.9$ implies

$$r_{\rm D} \gtrsim 0.05 \quad (95\%), \qquad (12)$$

and the further constraint that $F_{\rm NL} \lesssim -0.1$ yields

$$r_{\rm D} \gtrsim 0.62 \quad (95\%), \qquad (13)$$

indicating that the curvaton field has a non-negligible energy density when it decays.

- Our findings, therefore, pave the way to constrain inflation models with PTAs.

Equation of state of the early Universe

- The observed spectrum of SIGW per $\ln k$ today is

$$\Omega_{\mathrm{GW},0}h^2 \approx 1.62\times 10^{-5}\left(\frac{\Omega_{r,0}h^2}{4.18\times 10^{-5}}\right)\left(\frac{g_{*r}\left(T_{\mathrm{rh}}\right)}{106.75}\right)\left(\frac{g_{*s}\left(T_{\mathrm{rh}}\right)}{106.75}\right)^{-\frac{4}{3}}\Omega_{\mathrm{GW,rh}}. \tag{14}$$

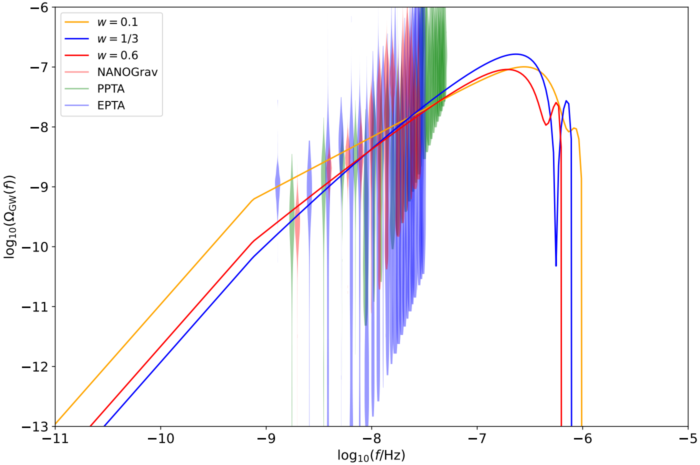
The SIGW spectrum for the scales $k \gtrsim k_{\mathrm{rh}}$ is

$$\Omega_{\mathrm{GW,rh}} = \left(\frac{k}{k_{\mathrm{rh}}}\right)^{-2b}\int_0^\infty dv\int_{|1-v|}^{1+v} du\, \mathcal{T}(u,v,w)\,\mathcal{P}_{\mathcal{R}}(ku)\mathcal{P}_{\mathcal{R}}(kv), \tag{15}$$

where $b \equiv (1-3w)/(1+3w)$. And $\Omega_{\mathrm{GW,rh}} \propto (k/k_{\mathrm{rh}})^2$ when $k \lesssim k_{\mathrm{rh}}$.

- The primordial power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2}\right). \tag{16}$$



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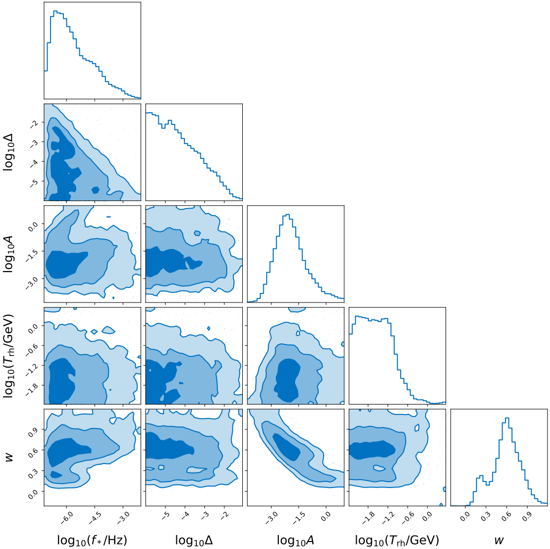

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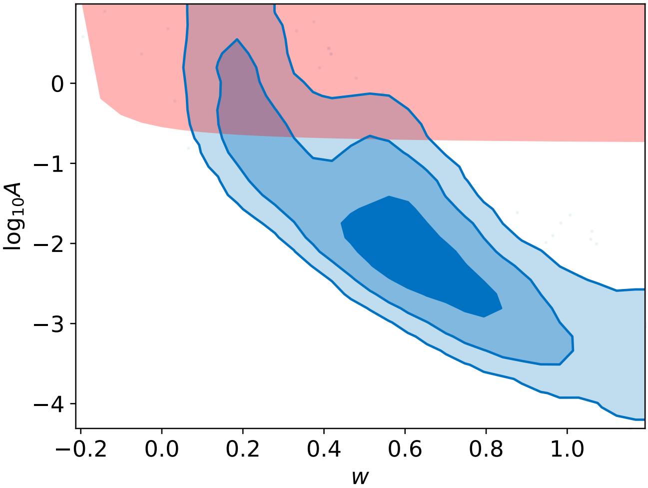
Implications

- Reheating temperature $T_{\rm rh} \lesssim 0.2\mathrm{GeV}$.
- $w < 0$ is excluded at 95% confidence level.
- $w = 1/3$ is consistent with the PTA data.
- w peaks at around 0.6.
- Since during the oscillation of inflaton, $w = \frac{p-2}{p+2}$ for an power-law potential $V(\phi) \propto \phi^p$, then, the constraint on w implies a ϕ^8 bottom of the inflationary potential.

Overcoming excessive PBH production



Summary

- PTAs are opening a new window in the nHz band.
- SIGWs can explain recent PTA signal.
- PTA can explore the nature of the early Universe through SIGWs, including
  - local-type non-Gaussianity of curvature perturbation
    Lang Liu, ZCC, Qing-Guo Huang, 2307.01102
  - equation of state of the early Universe
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  - $\cdots$