Constraints on Primordial-Black-Hole Population and Cosmic Expansion History from GWTC-3

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Outline

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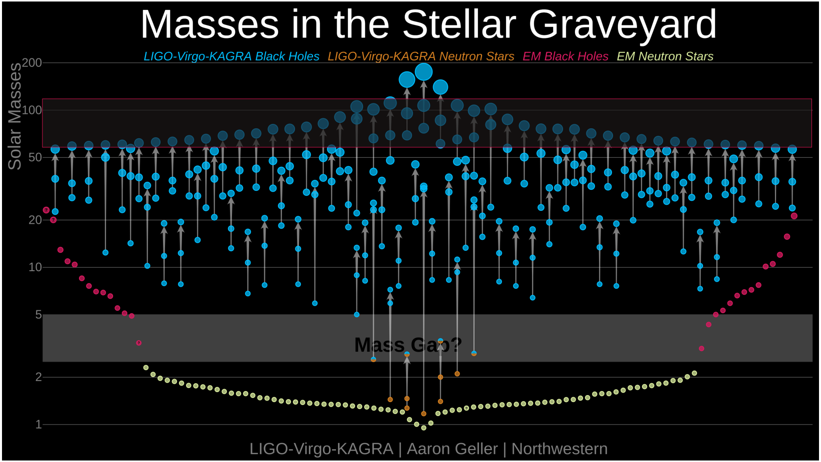


GWTC-3: 90 GW events $(2$ BNSs $+\ 3$ NSBHs $+\ 85$ BBHs$)$

What we know after LIGO-Virgo-KAGRA (LVK)

- There are many binary black holes (BBHs).
- They do have mass distribution.
- They can merge within Hubble time.

What we don't know after LVK

- Where do these BHs come from?
- What is the formation mechanism for these binaries?

The heavy BBHs, such as GW190521 with $m_1 = 85^{+21}_{-14} M_\odot$ and $m_2 = 66^{+17}_{-18} M_\odot$, challenge the astrophysical black hole (ABH) scenario.

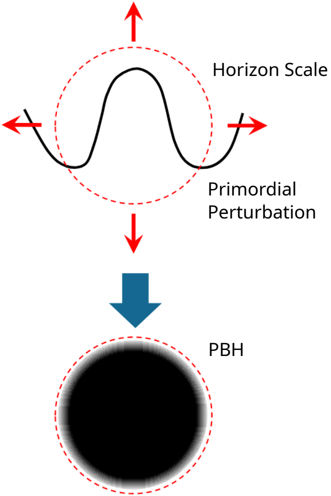
Primordial black hole?

Primordial black holes (PBHs)

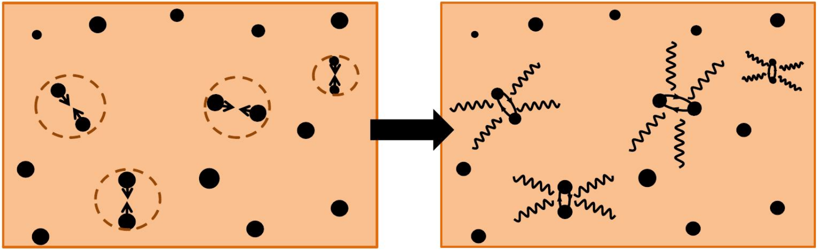
- PBHs are formed in the early universe by gravitational collapse of primordial density perturbations *Carr, Hawking, MNRAS (1974)*
- PBH mass can span many orders

$$m_{\rm PBH} \sim \frac{t}{G} \sim 10^{-18} \left(\frac{t}{10^{-23} s}\right) M_\odot \quad (1)$$

- PBHs survived from Hakwing radiation can be DM candidates.
- PBHs can explain LVK BBHs.



Formation of PBH binaries



- PBHs distributed randomly in the early Universe.
- Two neighboring PBHs decouple from the expansion background due to gravitational interaction and form a bound system.
- The momentum provided by other PBHs and linear density perturbations prevent the binary from head-on colliding.
- PBH binaries coalescence due to GW radiation and will be detected by LVK.

Dynamics of a PBH binary

- Equation of motion

$$\ddot{r} - \left(\dot{H} + H^2\right) r + \frac{m_b}{r^2}\frac{r}{|r|} = 0, \quad m_b = m_i + m_j. \tag{2}$$

- Semi-major axis a of the formed binary

$$a = \frac{0.1\bar{x}}{f_b} X^{\frac{4}{3}}, \quad X \equiv x^3/\bar{x}^3. \tag{3}$$

- Torques by all of other PBHs and density perturbations

$$j_X \approx 0.5\left(f^2 + \sigma_{\rm eq}^2\right)^{1/2} \frac{X}{f_b}, \quad f_b = f_i + f_j. \tag{4}$$

- Coalescence time *Peters, Phys. Rev. (1964)*

$$t_c = \frac{3}{85}\frac{a^4}{m_i m_j m_b} j^7. \tag{5}$$

Merger Rate Density

$$\begin{aligned}\mathcal{R}_{12}(t) \approx \quad & 2.8 \cdot 10^6 \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} f_{\rm pbh}^2 (0.7 f_{\rm pbh}^2 + \sigma_{\rm eq}^2)^{-\frac{21}{74}} \\ & \times \min\left(\frac{P(m_1)}{m_1}, \frac{P(m_2)}{m_2}\right) \left(\frac{P(m_1)}{m_1} + \frac{P(m_2)}{m_2}\right) \\ & \times (m_1 m_2)^{\frac{3}{37}} (m_1 + m_2)^{\frac{36}{37}}\end{aligned}$$

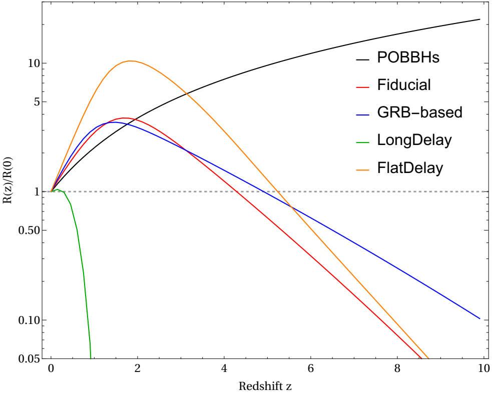
Zu-Cheng Chen, Qing-Guo Huang, APJ (2018)

- The fraction of PBHs in CDM is $f_{\rm pbh} \equiv \Omega_{\rm pbh}/\Omega_{\rm CDM}$.
- $\sigma_{\rm eq}^2 \sim 0.005^2$ is the variance of density perturbations of the rest DM.
- $P(m)$ is the mass function (PDF)

$$\int_0^\infty P(m) dm = 1.$$

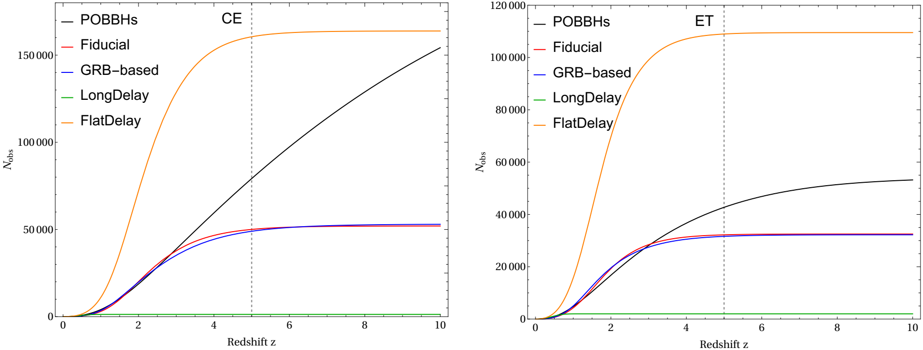
Distinguish PBHs from ABHs

- Subsolar mass BHs must be PBHs.
- High redshift BHs must be PBHs.
- Redshift evolution of merger rate



Distinguish PBHs from ABHs

$$N_{\rm obs}(z) = \int \mathrm{d}m_1 \mathrm{d}m_2 \int_0^z \mathcal{R}_{12}(z') \frac{\mathrm{d}VT}{\mathrm{d}z'} \mathrm{d}z'$$

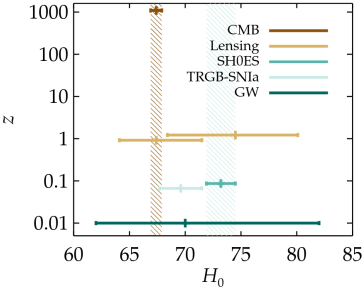


Zu-Cheng Chen, Qing-Guo Huang, JCAP (2020)

Hubble parameter $H(z)$

Hubble parameter is a fundamental observable that may help unveil the nature of dark energy and test general relativity.

- Hubble tension (crisis) at $\gtrsim 5\sigma$
  - $H_0 = 67.36 \pm 0.54$ km s^{-1} Mpc^{-1} from Planck 2018
  - $H_0 = 73.30 \pm 1.04$ km s^{-1} Mpc^{-1} from SH0ES team



- GWs provide an independent probe of $H(z)$.

- GW experiments measure the luminosity distance D_{L} and redshifted masses $m_1^{\mathrm{det}}, m_2^{\mathrm{det}}$

$$m_i = \frac{m_i^{\mathsf{det}}}{1 + z\left(D_{\mathrm{L}}; H_0, \Omega_{\mathrm{m}}\right)} \tag{6}$$

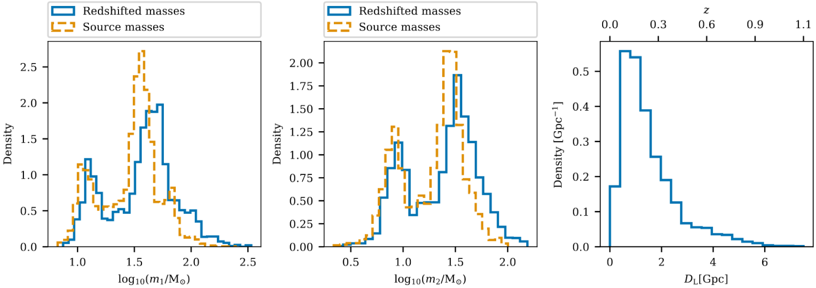
$$D_L(z) = \frac{(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{\mathrm{m}}(1+z')^3 + (1-\Omega_{\mathrm{m}})}} \tag{7}$$

- Standard siren: infer the redshift of the GW with electromagnetic counterparts, and directly constrain the cosmological parameters, such as GW170817.

Dark siren

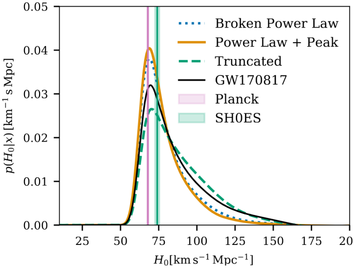
Even in the absence of electromagnetic observations, GWs alone can probe the expansion rate with the help of population properties, such as

- the peak of the mass distribution;
- the lower/upper mass cut-off;
- redshift distribution.



Masses and distance (redshift) distribution from GWTC-3.

- GWTC-3 contains ~ 2 times of GW events than GWTC-2
- LVK constrain the phenomenological ABH population and H_0 with GWTC-3 *LVK, arXiv:2111.03604*



- GWTC-3 (especially GW190521) is consistent with PBH scenario *Zu-Cheng Chen, Chen Yuan, Qing-Guo Huang, PLB (2022)*

Event	$R_{\rm LVK}[\mathrm{Gpc^{-3}\,yr^{-1}}]$	$R_{\rm PBH}[\mathrm{Gpc^{-3}\,yr^{-1}}]$	
		case I	case II
GW190521	$0.13^{+0.30}_{-0.11}$	$0.12^{+0.11}_{-0.07}$	$0.16^{+0.11}_{-0.08}$

- We will infer H_0 with PBH model using GWTC-3.

Population model

$$\mathcal{R}_{12}(t) \approx \quad 2.8 \cdot 10^6 \left(\frac{t(z)}{t_0}\right)^{-\frac{34}{37}} f_{\mathrm{pbh}}^2 (0.7 f_{\mathrm{pbh}}^2 + \sigma_{\mathrm{eq}}^2)^{-\frac{21}{74}} \\ \times \min\left(\frac{P(m_1)}{m_1}, \frac{P(m_2)}{m_2}\right)\left(\frac{P(m_1)}{m_1} + \frac{P(m_2)}{m_2}\right) \\ \times (m_1 m_2)^{\frac{3}{37}} (m_1 + m_2)^{\frac{36}{37}}$$

$$\mathcal{R}(\theta|\Phi) = R_0\, p(\theta|\Phi), \quad \theta = \{m_1, m_2, z\}, \quad \Phi \equiv \text{hyper parameter} \tag{8}$$

Local merger rate R_0

$$R_0 = \int_0^\infty \int_0^\infty \mathcal{R}(m_1, m_2, z = 0|\Phi) dm_1 dm_2 \tag{9}$$

Detector frame population probability

$$p_{\mathrm{pop}}(\theta|\Phi) = \frac{1}{1+z}\frac{dV_{\mathrm{c}}}{dz} p(\theta|\Phi) \tag{10}$$

Hierarchical Bayesian Inference

$$\boxed{\mathscr{L}(\mathbf{d}|\Phi) \propto N_{\exp}^{N_{\mathrm{obs}}} e^{-N_{\exp}} \prod_{i=1}^{N_{\mathrm{obs}}} \frac{\int \mathscr{L}(d_i|\theta)\, R_{\mathrm{pop}}(\theta|\Phi) d\theta}{\xi(\Phi)}} \tag{11}$$

- $\mathbf{d} = (d_1, \ldots, d_{N_{\mathrm{obs}}})$ are N_{obs} BBHs
- $\xi(\Phi)$ quantifies selection biases

$$\xi(\Phi) = \int P_{\mathrm{det}}(\theta)\, R_{\mathrm{pop}}(\theta|\Phi)\, \mathrm{d}\theta \approx \frac{1}{N_{\mathrm{inj}}} \sum_{j=1}^{N_{\mathsf{found}}} \frac{R_{\mathrm{pop}}(\theta_j|\Phi)}{p_{\mathrm{draw}}(\theta_j)}$$

where N_{inj} is the number of injections, N_{found} is the number of injections that are detected, and p_{draw} is the probability distribution from which the injections are drawn.

- $\mathscr{L}(d_i|\theta)$ is single event likelihood.

Bayes' theorem

$$\underbrace{p(H|D)}_{\text{posterior}} = \frac{\overbrace{p(D|H)}^{\text{likelihood}}\,\overbrace{p(H)}^{\text{prior}}}{\underbrace{p(D)}_{\text{normalization factor}}}$$

- Bayes factor

$$\mathrm{BF} = \frac{\operatorname{Pr}(\mathscr{D} \mid \mathscr{M}_1)}{\operatorname{Pr}(\mathscr{D} \mid \mathscr{M}_0)}$$

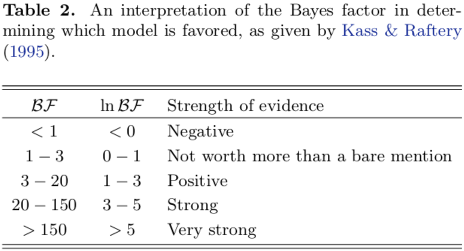


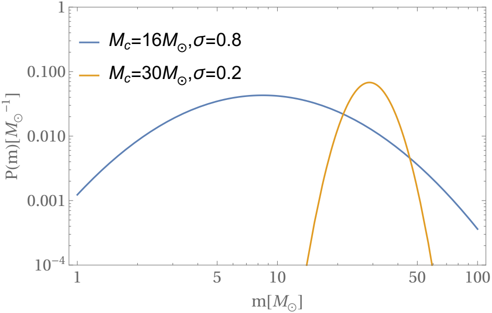
Table 2. An interpretation of the Bayes factor in determining which model is favored, as given by Kass & Raftery (1995).

$\mathcal{BF}$	$\ln\mathcal{BF}$	Strength of evidence
< 1	< 0	Negative
$1 - 3$	$0 - 1$	Not worth more than a bare mention
$3 - 20$	$1 - 3$	Positive
$20 - 150$	$3 - 5$	Strong
> 150	> 5	Very strong

Lognormal PBH mass function

$$P(m,\sigma_c,M_c) = \frac{1}{\sqrt{2\pi}\sigma_c m}\exp\left(-\frac{\ln^2(m/M_c)}{2\sigma_c^2}\right) \qquad (12)$$

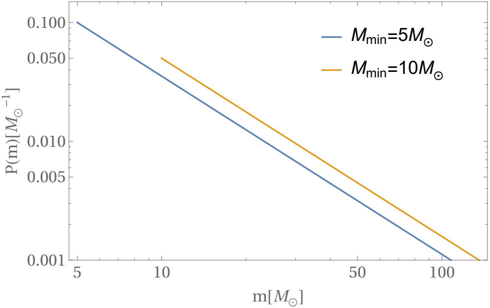
- Associate with power spectra with a smooth symmetric peak.
- M_c and σ_c are the peak and width of the mass spectrum.
- $\Phi = \{H_0, \Omega_m, \sigma_c, M_c\}$



Power-law PBH mass function

$$P(m, M_{\min}) = \frac{1}{2} M_{\min}^{1/2} m^{-3/2} \Theta(m - M_{\min}) \tag{13}$$

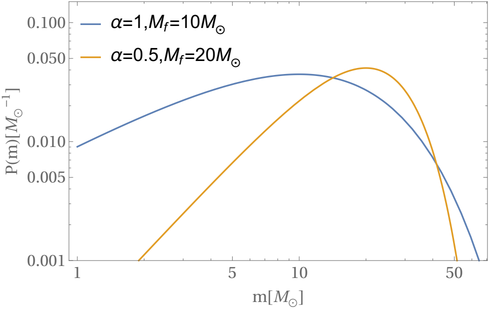
- Associate with a broad or flat power spectrum.
- $M_{\min}$ is the lower mass cut-off.
- $\Phi = \{H_0, \Omega_{\rm m}, M_{\min}\}$



Critical collapse (CC) PBH mass function

$$P(m, \alpha, M_{\mathrm{f}}) = \frac{\alpha^2\, m^{\alpha}}{M_{\mathrm{f}}^{1+\alpha}\, \Gamma(1/\alpha)} \exp\left(-(m/M_{\mathrm{f}})^{\alpha}\right) \tag{14}$$

- Associate with a monochromatic power spectrum.
- With an upper cut-off $\mathscr{O}(M_{\mathrm{f}})$, but no lower mass cut-off.
- $\Phi = \{H_0, \Omega_{\mathrm{m}}, \alpha, M_{\mathrm{f}}\}$



Parameter	**Description**	**Prior**
	Merger rate evolution	
R_0	Local merger rate of PBH binaries in $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$.	$\mathcal{U}(0, 200)$
	Cosmological parameters	
H_0	Hubble constant in $\mathrm{km\,s}^{-1}\mathrm{Mpc}^{-1}$.	$\mathcal{U}(10, 200)$ (Wide prior) $\mathcal{U}(65, 77)$ (Restricted prior)
Ω_m	Present-day matter density of the Universe.	$\mathcal{U}(0, 1)$ (Wide prior) $\delta(0.315)$ (Restricted prior)
	Lognormal PBH mass function	
M_c	Peak mass in $M_\odot$.	$\mathcal{U}(5, 50)$
σ_c	Mass width.	$\mathcal{U}(0.1, 2)$
	Power-law PBH mass function	
M_min	Lower mass cut-off in $M_\odot$.	$\mathcal{U}(3, 10)$
	Critical collapse (CC) PBH mass function	
M_f	Horizon mass scale in $M_\odot$.	$\mathcal{U}(5, 50)$
α	Universal exponent.	$\mathcal{U}(0.5, 5)$

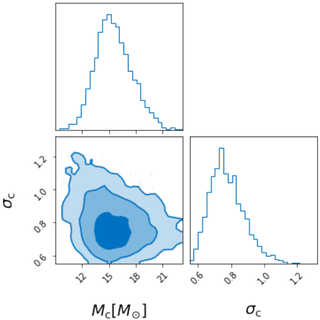
PBH mass model	$\log_{10} \mathscr{B}$
Lognormal	2.99
Power-law	0
CC	3.12

Table: $\log_{10}$ Bayes factor between different mass models and the Power-law mass model, for the case of a flat ΛCDM cosmology with wide priors. Power-law PBH mass model is strongly disfavored.

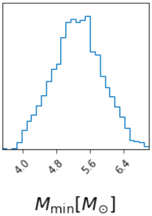
PBH mass model	$\log_{10} \mathscr{B}$
Lognormal	-0.02
Power-law	-0.11
CC	0.20

Table: $\log_{10}$ Bayes factor comparing runs that adopt the same PBH mass model but different cosmologies: Wide priors versus Restricted priors. No evidence in favor of any of these two cosmological models.

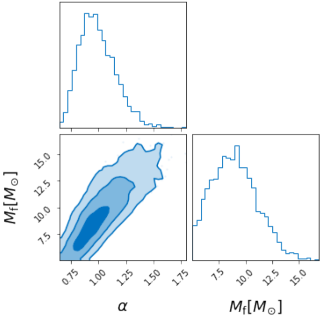
Lognormal



power-law

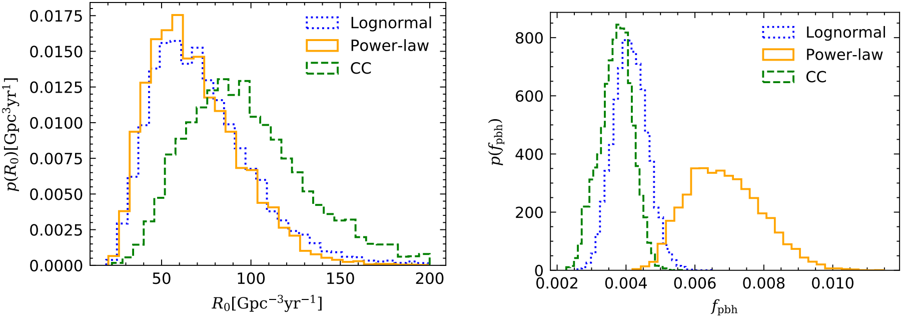


CC



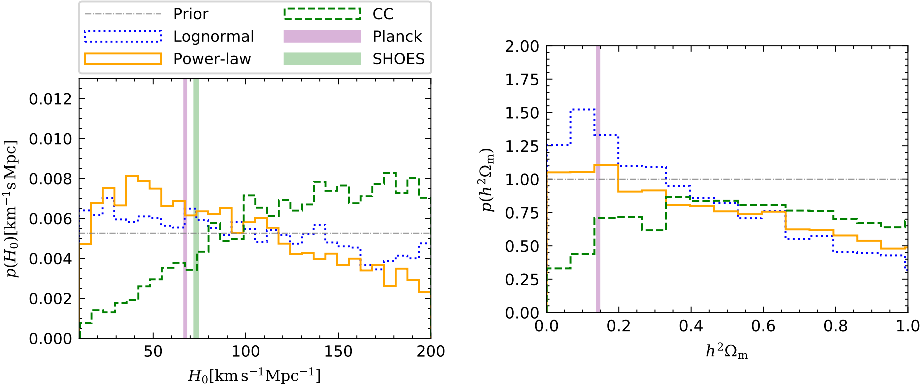
The PBH mass functions are well constrained.

Local merger rate and f_{pbh}



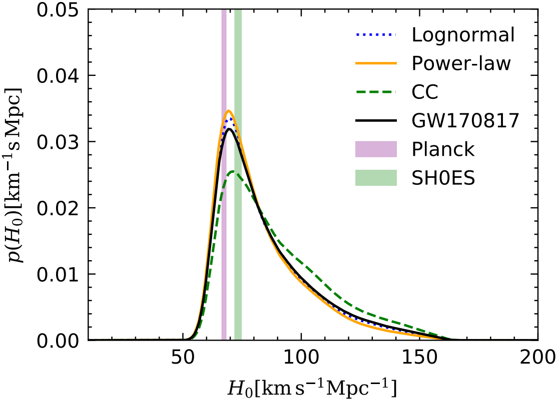
	Lognormal	Power-law	CC
$R_0[\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}]$	69^{+31}_{-22}	65^{+30}_{-21}	93^{+37}_{-29}
$f_{\mathrm{pbh}}/10^{-3}$	$4.1^{+0.5}_{-0.8}$	$6.8^{+1.2}_{-1.0}$	$3.7^{+0.4}_{-0.5}$

The stellar-mass PBHs cannot dominate CDM.



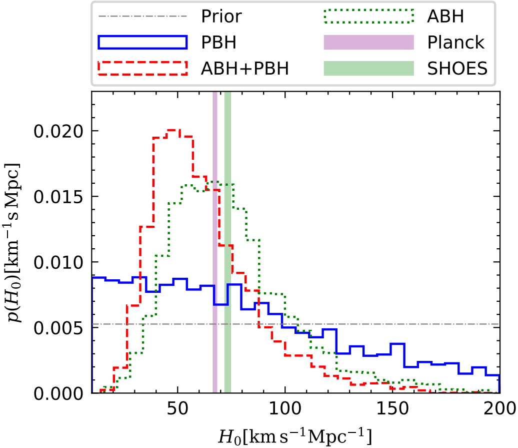
- The constraints on cosmological parameters are weak and informative.

Combined with GW170817



	Lognormal	Power-law	CC	ABH
H_0[km s^{-1} Mpc^{-1}]	69^{+19}_{-8}	69^{+19}_{-8}	70^{+26}_{-8}	68^{+12}_{-8}

ABH+PBH



The ABH+PBH model can better describe the mass distribution, thus improving the precision to constrain the Hubble constant.

Conclusions

- PBH mass distribution can be well constrained.
- The constraints on standard $\Lambda\mathrm{CDM}$ cosmological parameters are rather weak and in agreement with current results.
- When combining with GW170817, the Hubble constant H_0 is constrained to be $69^{+19}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $70^{+26}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ for the lognormal and critical collapse mass models, respectively.
- With increased BBH events, the mixed ABH+PBH model can provide a robust statistical inference for both the population and cosmological parameters.